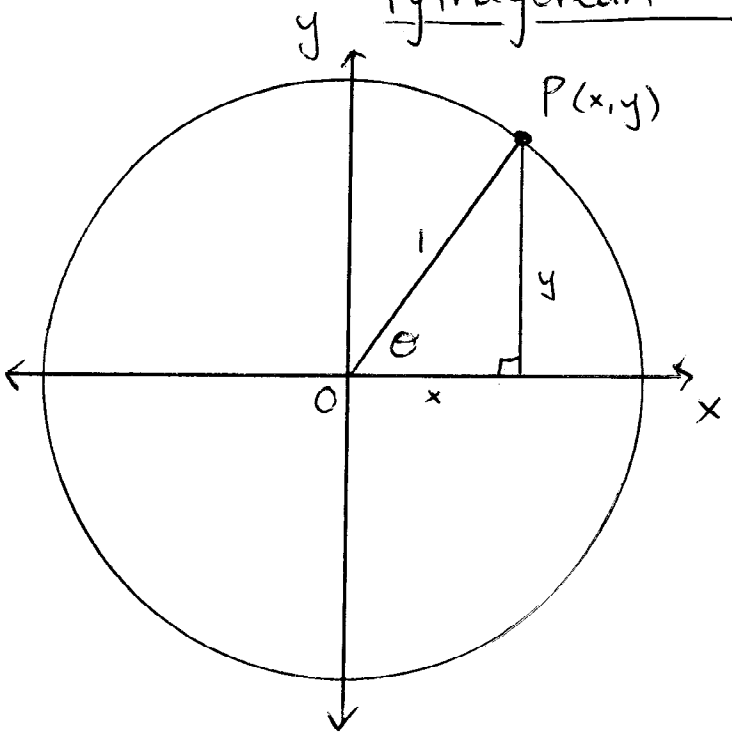
Pythagorean Identities



P is a point on the unit circle, ∴ $|OP| = 1$ unit.

If θ is some angle between the positive x-axis and OP, then:

$$\sin\theta = \frac{y}{1}, \quad \cos\theta = \frac{x}{1}, \quad \tan\theta = \frac{y}{x}$$

Pythagorean thm: $x^2 + y^2 = c^2$

from unit circle: $(\cos\theta)^2 + (\sin\theta)^2 = 1^2$

$$\therefore \sin^2\theta + \cos^2\theta = 1 \text{ and}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Prove: $\sec^2\theta = 1 + \tan^2\theta$

L.S $= \sec^2\theta$ R.S $= 1 + \tan^2\theta$

$$= \frac{1}{\cos^2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}$$

$$= \tan^2\theta + 1$$

L.S = R.S

Prove: $1 + \cot^2\theta = \csc^2\theta$

L.S $= 1 + \cot^2\theta$ R.S $= \csc^2\theta$

$$\text{R.S} = \frac{1}{\sin^2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta}$$

$$= \frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta}$$

$$= 1 + \cot^2\theta$$

∴ L.S = R.S

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \cot^2\theta = \csc^2\theta$$
$$1 + \tan^2\theta = \sec^2\theta$$