

1. Match the equation of each function with its graph.

(a)  $y = x^3 - x^2$

(b)  $y = x^4 - x^2$

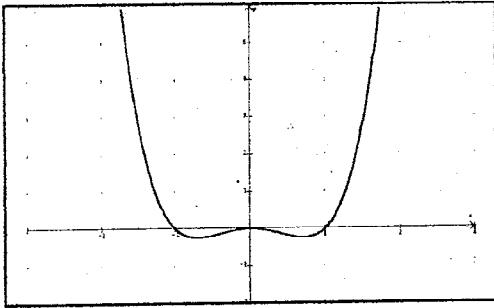
(c)  $y = -x^4 + x^3$

(d)  $y = \frac{1}{x^2 + 2}$

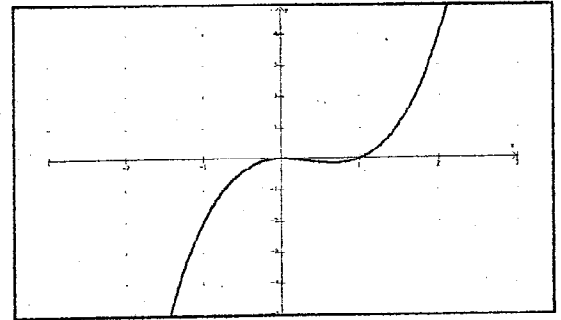
(e)  $y = \frac{1}{x^2 - 2}$

(f)  $y = \frac{x^2 + 2}{x^2 - 2}$

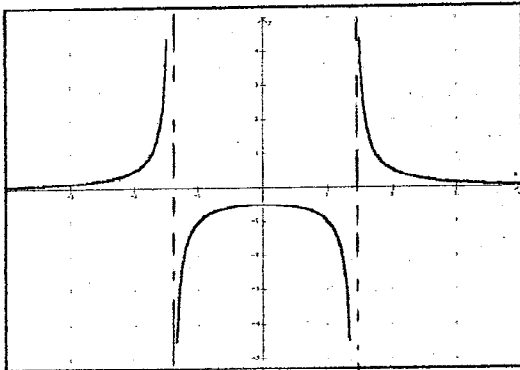
(i)



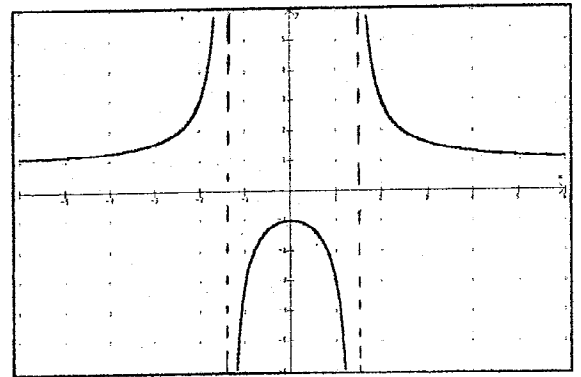
(ii)



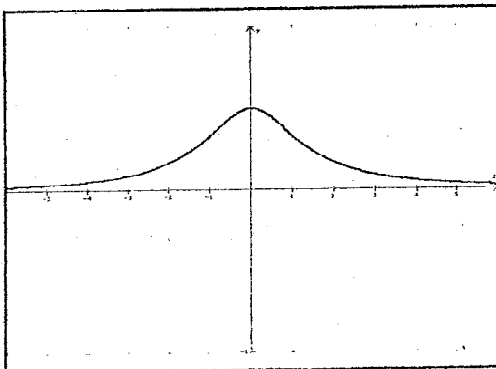
(iii)



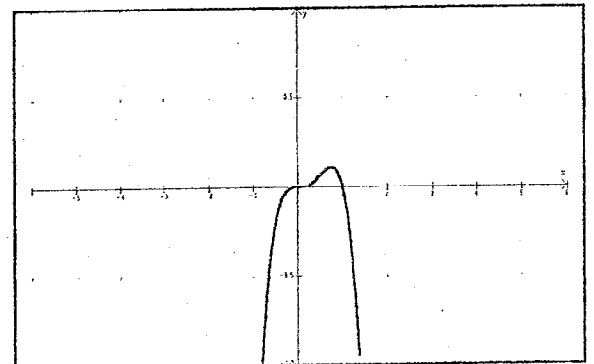
(iv)



(v)



(vi)



2. State the domain and range of each of the following functions:

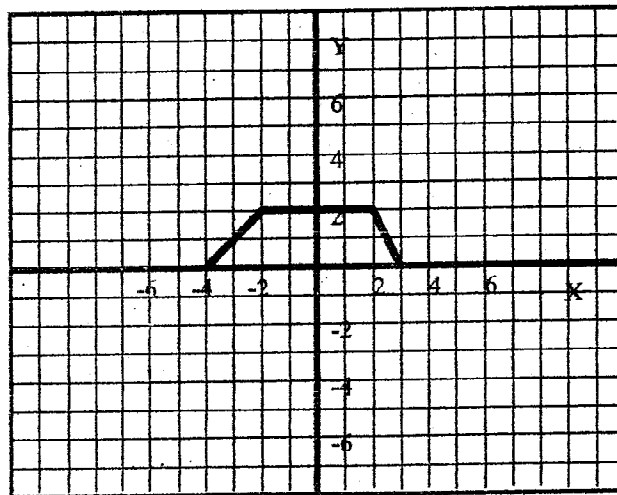
(a)  $f(x) = 3(x-2)^2 - 5$

(b)  $g(x) = \sqrt{x+5} + 3$

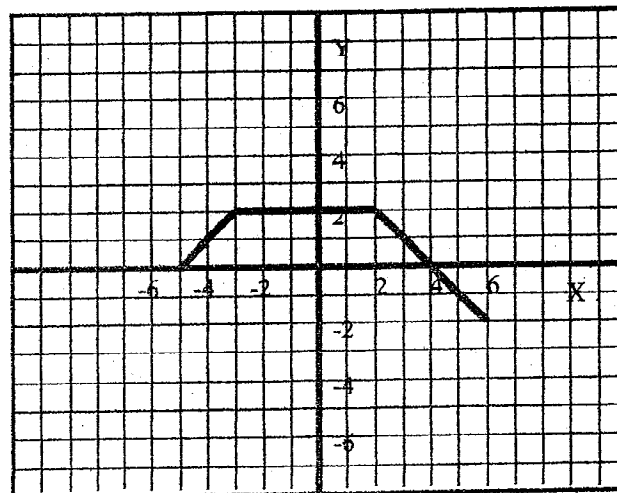
3. Is the relation  $4x^2 + y^2 = 16$  a function? Explain.

4. Given below is the graph of  $y = p(x)$ .

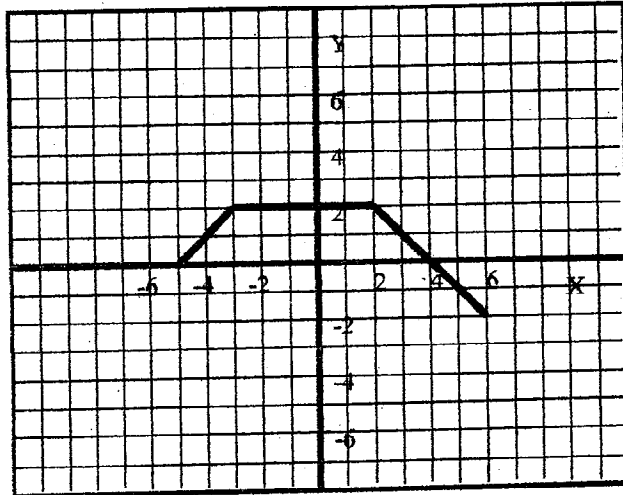
(a) Sketch the graph of  $y = -2p(0.5x + 1) - 4$ .



(b) Sketch the graph of  $y = p^{-1}(x)$ . Is this relation a function? Explain.



- (c) Sketch the graph of  $y = \frac{1}{p(x)}$ .



5. Sketch the graphs of  $y = \sin x$  and  $y = \cos x$  for  $x \in [0, 2\pi]$ .  
Now sketch the graph of  $h(x) = \sin x + \cos x$  for  $x \in [0, 2\pi]$ .
6.  $f(x) = \sqrt{x+3}$ . Determine  $f^{-1}(x)$ .
7.  $q(x) = 3x + 4$  and  $w(x) = x + 1$ . Determine  $w(x) + q^{-1}(x)$ .
8. Given:  $f(x) = 3x - 4$  and  $g(x) = x^2 - 5$   
State  $(g \circ f)(x)$  and simplify.
9. A cubic function has roots  $-3, 2$  and  $4$ . If its graph passes through the point  $(3, -1)$ , find its equation.
10. The difference table for a function is created. The third difference is  $48$ . The  $y$ -intercept of the function is  $3$  and its  $x$ -intercept is  $2$ . Another point on the graph of the function is  $(1, 10)$ . Find the equation of the function.

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11. State the remainder when  $x^5 - x^3 + 1$  is divided by  $x - 3$ .
12. Factor:  $2x^3 - x + 1$ .
13. Solve for  $x$ :  $x^3 - 27 = 0$ . Include any complex roots.
14. Solve:  $x^2 + x > 6$ .
15. The area of a circular oil spill is given by the formula  $A = \pi r^2$ .
- (a) Determine the average rate of change of the area with respect to the radius when the radius grows from 1 km to 3 km.
- (b) Determine the rate of change of the area with respect to the radius when the radius is 2 km.
16. Determine the equation of the tangent to the curve  $f(x) = x - x^2$  when  $x = 1$ . Use two different methods.
17. Suppose the graph of the function  $y = s(x)$  is drawn. The actual equation is not given. How would you use the graph to determine the rate of change of  $y$  with respect to  $x$  at  $x = a$ ?
18. Graph the function  $p(x) = \frac{x^2 - 4}{9 - x^2}$  by first determining
- (a) if the function is even, odd or neither even nor odd,
- (b) the equations of its asymptotes,
- (c) how the graph approaches these asymptotes, and
- (d) its intercepts.
19. Solve:  $\frac{x - 5}{x + 1} + \frac{1}{x} = \frac{x - 3}{x}$ . State all restrictions on  $x$ .

20. Solve:  $\frac{1}{x+1} < \frac{2}{x}$
21. Change  $140^\circ$  to exact radian measure.
22. Change to degree measure:
- (a)  $\frac{10\pi}{6}$  rad (b) 4 rad
23. Give two positive and two negative angles coterminal with an angle of  $\frac{-\pi}{3}$  rad
24.  $P(-1, \sqrt{3})$  is on the terminal arm of an angle,  $\theta$ , in standard position.
- (a) Determine all six trigonometric ratios of  $\theta$ .
- (b) Determine  $\theta$  in radian measure.
25. Evaluate exactly:  $\cos \frac{3\pi}{4} - \sin \frac{7\pi}{6}$
26. Sketch the graph of  $y = -2\cos(3\theta - \pi) + 4$  for one cycle. Also state the amplitude, period and phase shift.
27. A sinusoidal function has amplitude 4, period  $\pi$  and phase shift  $\frac{\pi}{2}$ . Give the equations of two different functions which have these properties:
28. Prove the following identities:
- (a)  $\tan \theta + \frac{1}{\tan \theta} = \frac{1}{\sin \theta \cos \theta}$  (b)  $\frac{\csc \theta}{1 + \csc \theta} + \frac{\csc \theta}{1 - \csc \theta} = -\frac{2 \sin \theta}{\cos^2 \theta}$
- (c)  $\frac{\sin 2x}{1 + \cos 2x} = \tan x$
29. Find the exact value of  $\sin 75^\circ$ .

30. Solve for  $x$  where  $0 \leq x \leq 2\pi$ :

(a)  $\sin x = -0.5$

(b)  $\cos^2 x - 1 = 0$

(c)  $\tan 2x = 1$

(d)  $\csc x = -\frac{2}{\sqrt{3}}$

(e)  $2\sin^2 x = 1 - \sin x$

(f)  $\sin 2x = \sin x$

31. The latitude on the earth's surface when the maximum height of a tide is obtained is given by the roots of the equation  $4\cos^2\theta - 3 = 0$ . Find the latitudes.

32. Sketch the graph of  $y = 2^{x^3} - 4$ .

33. Sketch the graph of  $y = -\log_2(4x + 4)$ .

34. Change to logarithmic form:  $y = 6^7$ .

35. Change to exponential form:  $a = \log_3 p$ .

36. Evaluate  $\log_3 0.6$  correct to 3 decimal places.

37. Evaluate each of the following without using your calculator:

(a)  $\log_5 \sqrt[4]{125}$

(b)  $\log_6 18 - \log_6 2 + \log_6 24$

(c)  $6^{\frac{1}{2} \log_6 49}$

(d)  $2 \log_4 8$

(e)  $\frac{\log_6 125}{\log_6 5}$

(f)  $\log_2 \left( \sec \frac{\pi}{3} \right)$

38. Solve for  $x$ ,  $x \in \mathfrak{R}$  :

(a)  $4 \log_2 x^2 = 8$

(b)  $2 \log_{10}(x - 1) = \log_{10}(x + 5)$

(c)  $\log x^5 = 20$

(d)  $27^{x+5} = 9^{2x}$

(e)  $25^{x-4} = 125^x$

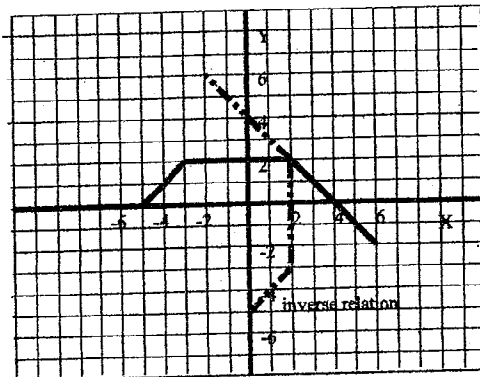
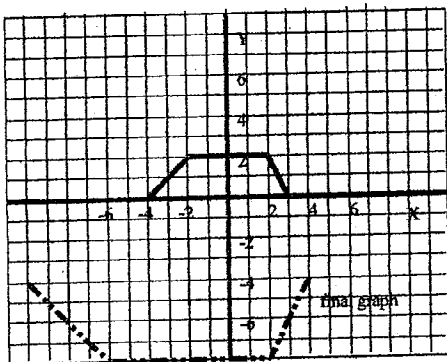
(f)  $4^x = 15$

39. The number of bacteria doubles every 30 minutes. Initially there are 100 bacteria. After how many hours are there 6 553 600 bacteria?

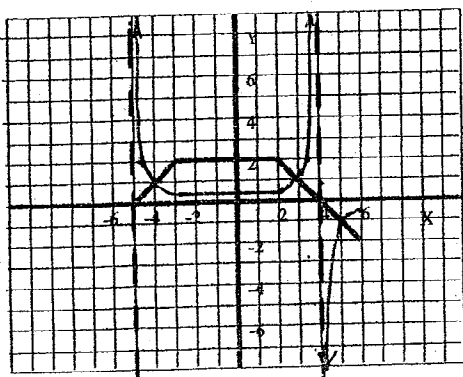
40. Solve for  $x$  correct to one decimal place:  $25 = 128 \times 10^{-0.016x}$

Answers

1. (a) (ii), (b) (i), (c) (vi), (d) (v), (e) (iii), (f) (iv)  
 2. (a) domain =  $\mathfrak{R}$ , range is  $y \in [-5, \infty)$  (b) domain is  $x \in [-5, \infty)$ , range is  $y \in [3, \infty)$   
 3. No! For  $x = 0$  in the domain, there are two values of  $y$  in the range,  $\pm 4$ .  
 4. (a) (b)

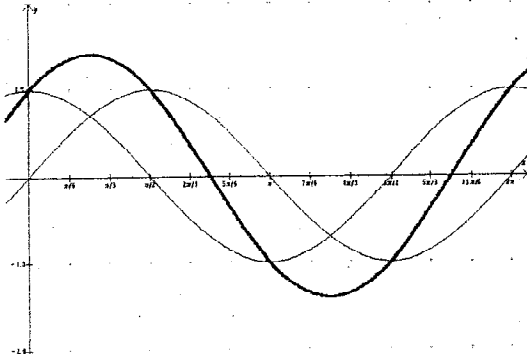


(c)



not a function! It fails the vertical line test.

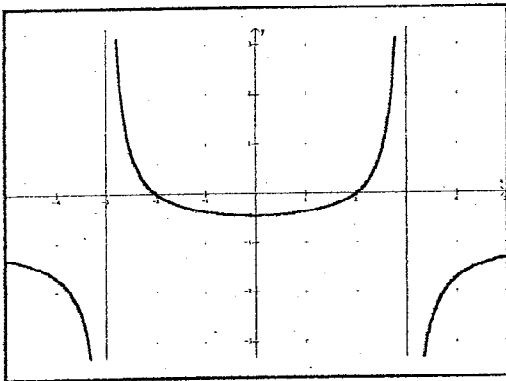
5.



15. (a)  $8\pi \text{ km}^2/\text{km}$  (b)  $4\pi \text{ km}^2/\text{km}$

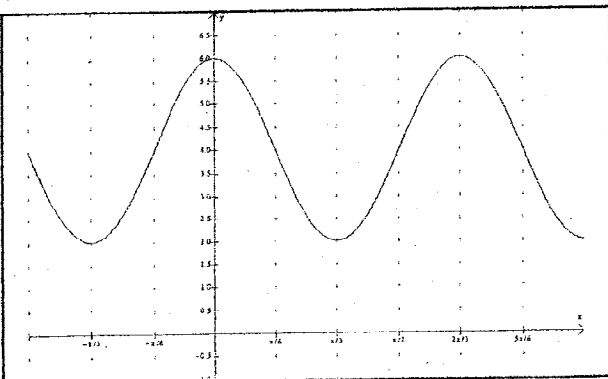
17. Draw an approximate tangent at the point  $(a, s(a))$  and estimate its slope from the graph.

18.



25.  $\frac{1 - \sqrt{2}}{2}$

26. Amplitude = 2, Period =  $\frac{2\pi}{3}$ , Phase shift =  $\frac{\pi}{3}$



6.  $f^{-1}(x) = x^2 - 3, x \geq 0$       7.  $\frac{4x - 1}{3}$

8.  $(g \circ f)(x) = 9x^2 - 24x + 11$       9.  $y = \frac{1}{6}(x+3)(x-2)(x-4)$

10.  $y = 8x^3 - \frac{65}{2}x^2 + \frac{63}{2}x + 3$

11. 217

12.  $(x+1)(2x^2 - 2x + 1)$

13. 3,  $\frac{-3 \pm 3i\sqrt{3}}{2}$

14.  $x \in (-\infty, -3) \text{ or } (2, \infty)$

16.  $y = -x + 1$

19.  $x = 2, x \neq -1, 0$

20.  $x \in (-2, -1) \text{ or } (0, \infty)$

21.  $\frac{7\pi}{9}$

22. (a)  $300^\circ$  (b) approx.  $229.2^\circ$

23.  $\frac{5\pi}{3}, \frac{11\pi}{3}, \frac{-7\pi}{3}, \frac{-13\pi}{3}$

24. (a)  $\sin \theta = \frac{\sqrt{3}}{2}, \csc \theta = \frac{2}{\sqrt{3}}$  (b)  $\frac{2\pi}{3}$

$\cos \theta = \frac{-1}{2}, \sec \theta = -2$

$\tan \theta = -\sqrt{3}, \cot \theta = \frac{-1}{\sqrt{3}}$

27. e.g.,  $y = 4\sin(2x - \pi), y = 4\cos(2x - \pi)$

29.  $\frac{\sqrt{3} + 1}{2\sqrt{2}}$

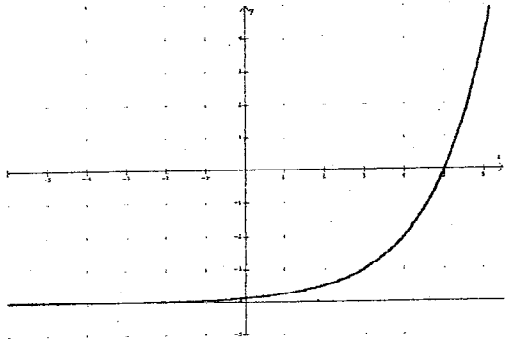


30. (a)  $\frac{7\pi}{6}, \frac{11\pi}{6}$  (b)  $0, \pi, 2\pi$  (c)  $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$  (d)  $\frac{5\pi}{6}, \frac{7\pi}{6}$

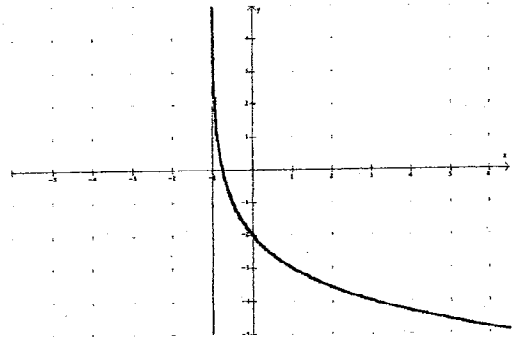
(e)  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$  (f)  $0, \pi, 2\pi, \frac{\pi}{3}, \frac{5\pi}{3}$

31.  $30^\circ, 150^\circ, 210^\circ, 330^\circ$

32.



33.



34.  $7 = \log_6 y$  35.  $p = 3^a$  36.  $-0.465$  37. (a)  $\frac{3}{4}$  (b) 3 (c) 7 (d) 3 (e) 3 (f) 1

38. (a)  $\pm 2$  (b) 4 (c) 10 000 (d) 15 (e) -8 (f) approx. 1.95 39. 8 h 40. 44.3