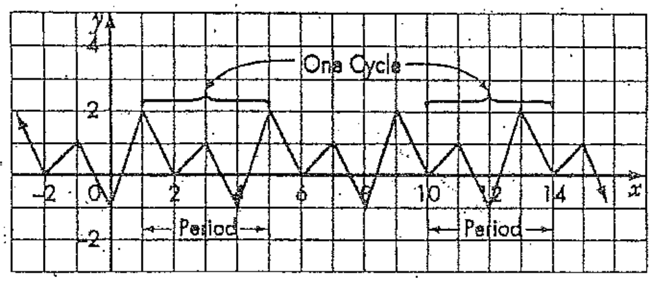
Graphs of the Primary Trigonometric Functions

Text: Section 5.1 & 5.3

Periodic Functions: *A function has a pattern of y-values that repeats at regular intervals.*



One complete pattern of a periodic function is called ***a cycle.***

The ***period*** *is the horizontal distance from the beginning of one cycle to the beginning of the next cycle.*

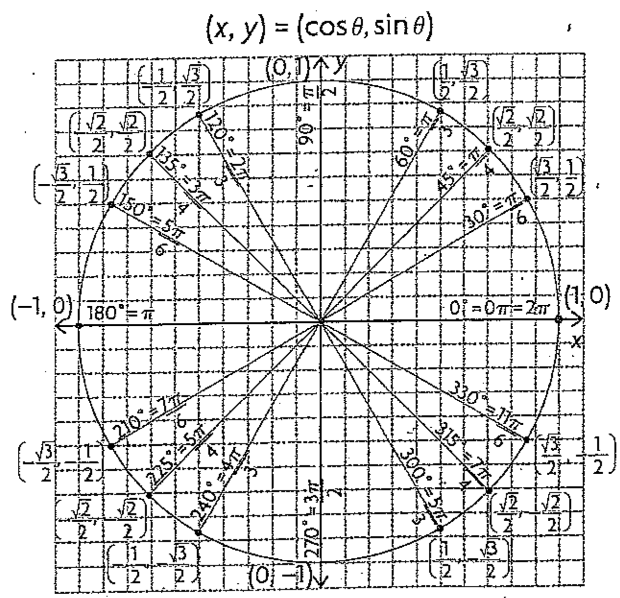
The ***amplitude*** *of a periodic function is half the difference between the maximum value of the function and the minimum value of the function. (Note: Amplitude is always a* ***positive*** *value.)*

$$\text{Amplitude} = \frac{\text{max value} - \text{min value}}{2}$$

The ***phase shift*** *(or* ***phase angle****) is a horizontal translation of a trigonometric function.*

Each point on the unit circle that is centred at the origin can be represented by the ordered pair

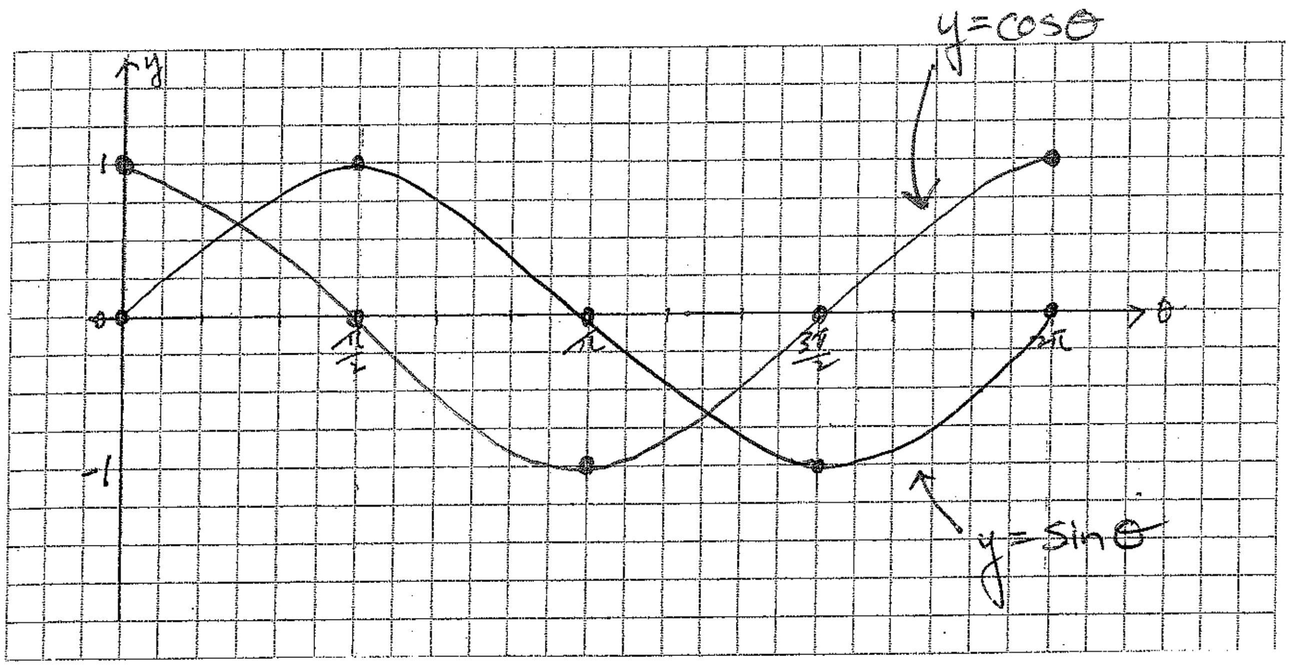
$(x, y) = (\cos\theta, \sin\theta)$.



Sketching the Sine and Cosine function using a 5-point method

This method uses the fact that one cycle of a sine or cosine function includes 5 points involving **maximum (1)**, **minimum (-1)**, and **zeros**. These 5 key points are equally apaced along the x-axis, so they divide the **period** into **quarters**. The graph of a **sine** function is plotted below using the
5-point method.

Angle	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y = Sin θ	**0**	**1**	**0**	**-1**	**0**
y = Cos θ	**1**	**0**	**-1**	**0**	**1**



Consider y = Sin θ.

State: Period = 2π Amplitude = 1

Maximum Value = 1 Minimum Value = -1

Domain = $\{x \mid x \in \mathbb{R}\}$ Range = $\{y \mid -1 \leq y \leq 1\}$

Sketching Functions of the Form $f(x) = \mathbf{a}\ \text{Sin}\ [\mathbf{k}(x - \mathbf{d})] + \mathbf{c}$

and $f(x) = \mathbf{a}\ \text{Cos}\ [\mathbf{k}(x - \mathbf{d})] + \mathbf{c}$.

where **a** is the **amplitude**

period = $\frac{2\pi}{k}$, **k** can be also considered as the **number of cycles in** 2π **radians.**

d is the **phase shift** with respect to $y = a\ \text{Sin}\ \mathbf{k}\theta$ or $y = a\ \text{Cos}\ \mathbf{k}\theta$

c is the **vertical shift.**

Review:

When using transformations, perform them in the following order:

1) ***expansions/compression***
2) ***reflection***
3) ***translation***

Example #1. Sketch $y = 3\ \text{Cos}\ (2x + \frac{\pi}{3})$ for $0 \le x \le 2\pi$

Solution: **Factor out** the **k (the coefficient** of the x-term)

$$y = 3\ \text{Cos}\ \mathbf{2}\ (x + \frac{\pi}{6})$$

Now state the amplitude = 3

the period = π → period $= \frac{2\pi}{k} = \frac{2\pi}{2} = \pi$

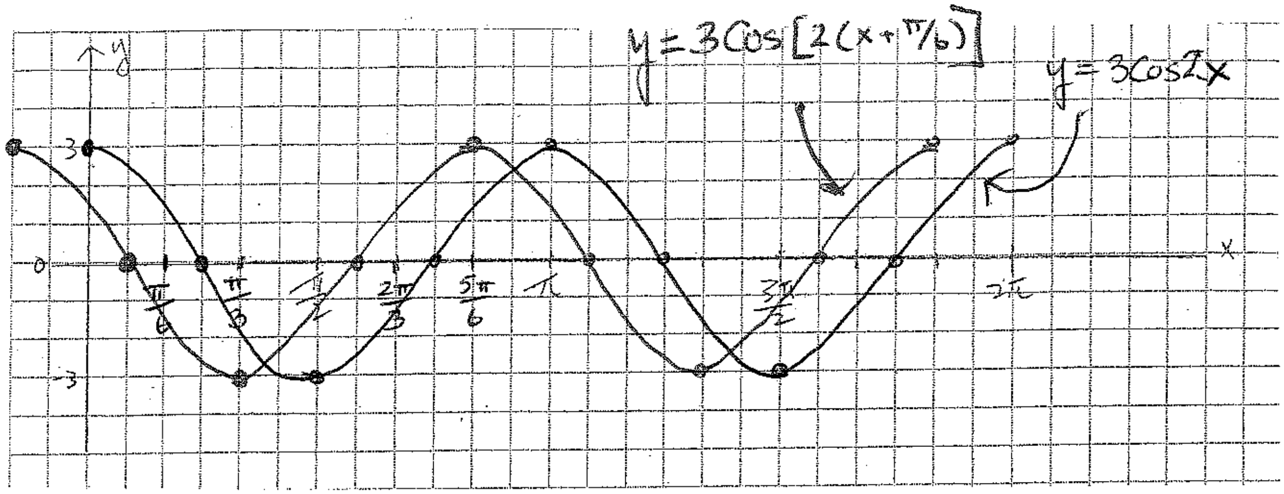
the phase shift = $\pi/6$ to the left.

Steps: 1) Divide the period (in this case, it is π radians) into 4 parts: $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$

2) Point one cycle of $y = 3\ \text{Cos}\ 2x$ using the 5-point method.

3) Continue the same pattern to complete the other cycle up to 2π radians.

4) Finish up the question by translating $y = 3\ \text{Cos}\ 2x$ by a **phase shift** of $\frac{\pi}{6}$ radians horizontally to the **left.**



Example #2. Complete the following table.

Function	Amplitude	Period	Phase Shift	Vertical Shift
$y = 2\ Cos\ 2\left(x - \frac{\pi}{2}\right) - 1$	2	$= \frac{2\pi}{k}$ $= 2\pi/2$ $= \pi$	$\pi/2$ to the right	1 unit down
$y = Sin\ (3x - \pi)$ $y = \sin 3(x - \pi/3)$	1	$= 2\pi/k$ $= 2\pi/3$	$\pi/3$ to the right	$\emptyset$

Example #3. Write the equation for the function with the given characteristics.

$$y = a \text{ Sin } k(x - d) + c$$
$$\text{or } y = a \text{ Cos } k(x - d) + c$$

Type	Sine	Cosine	Sine	Cosine
Amplitude	0.4	3	6	10
Period	$\frac{2\pi}{3}$	$\frac{\pi}{3}$	3π	π
Phase Shift	π left	none	$\frac{2\pi}{3}$ right	$-\frac{\pi}{6}$
Vertical Shift	None	4	-2	3
Equation	$y = 0.4\sin 3(x+\pi)$	$y = 3\cos 6x + 4$		

$y = 6\sin[2/3(x - \frac{2\pi}{3})] - 2$

$y = 10\cos[2(x + \pi/6)] + 3$

If you are having difficulties, follow these steps:

Amplitude 0.4, **a = 0.4,** *Period* $= \frac{2\pi}{3}$ *Phase shift* $= \pi$ *left*

$$\frac{2\pi}{k} = \frac{2\pi}{3} \qquad d = -\pi$$

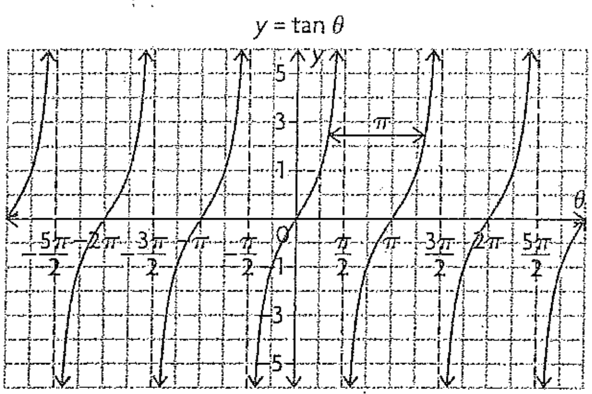
$$k = 3$$

Therefore the equation of the function is

$$y = a \sin k(x - d) + c$$

$$y = 0.4 \sin 3(x + \pi) + 0$$

The graph of $y = \text{Tan } x$



Period = π

amplitude = n/a

Maximum Value = n/a

Minimum Value = n/a

Vertical Asymptotes

$x = \frac{n\pi}{2}$, where n is an odd integer.

Domain = $\{x \mid x \neq \frac{n\pi}{2}\}$ where n is an odd integer.

Range = $\{y \mid y \in \mathbb{R}\}$.

Classwork/Homework

p.258 - p. 260

#9 - #19 (Omit #16, do it in math lab)

p.276 - p.278

#1 to 14, 21