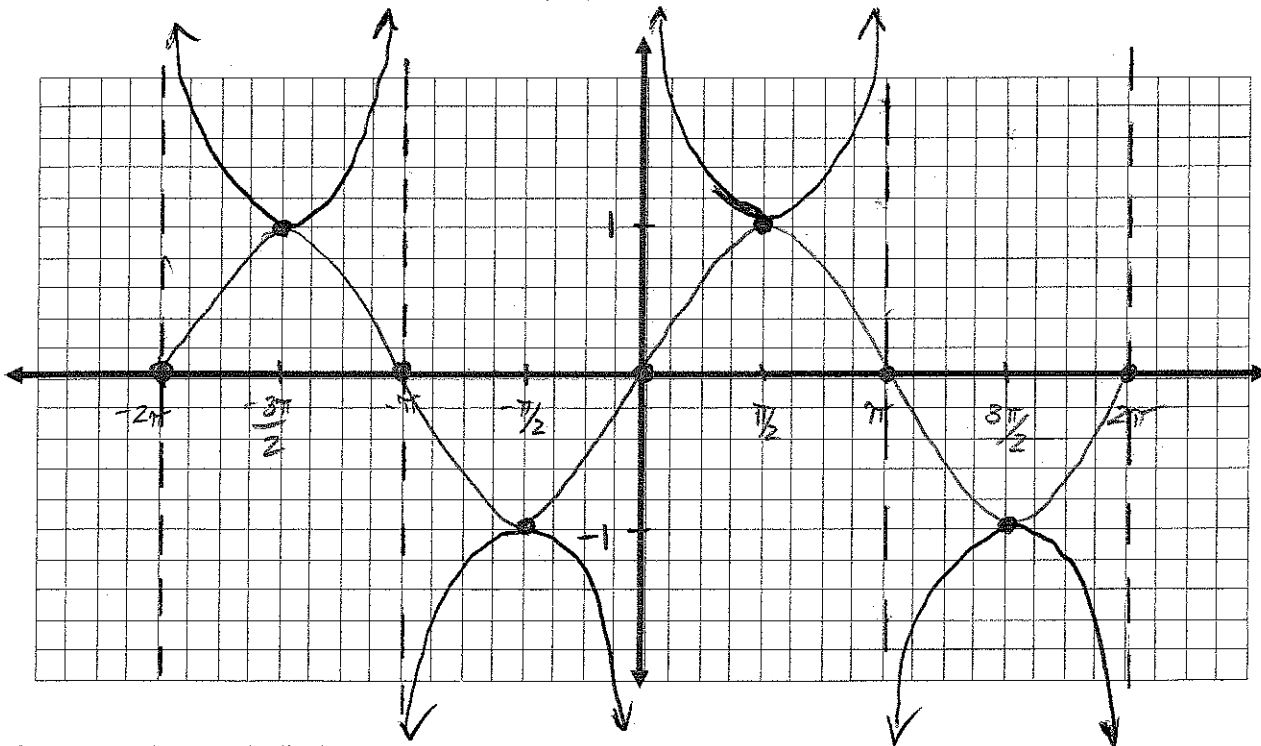## 5.2: Graphs of Reciprocal Trigonometric Functions

Recall:

➤ $y = \csc\theta$ is the reciprocal function of $y = \sin\theta$

➤ $y = \sec\theta$ is the reciprocal function of $y = \cos\theta$

➤ $y = \cot\theta$ is the reciprocal function of $y = \tan\theta$

Investigate: p. 261 – 262

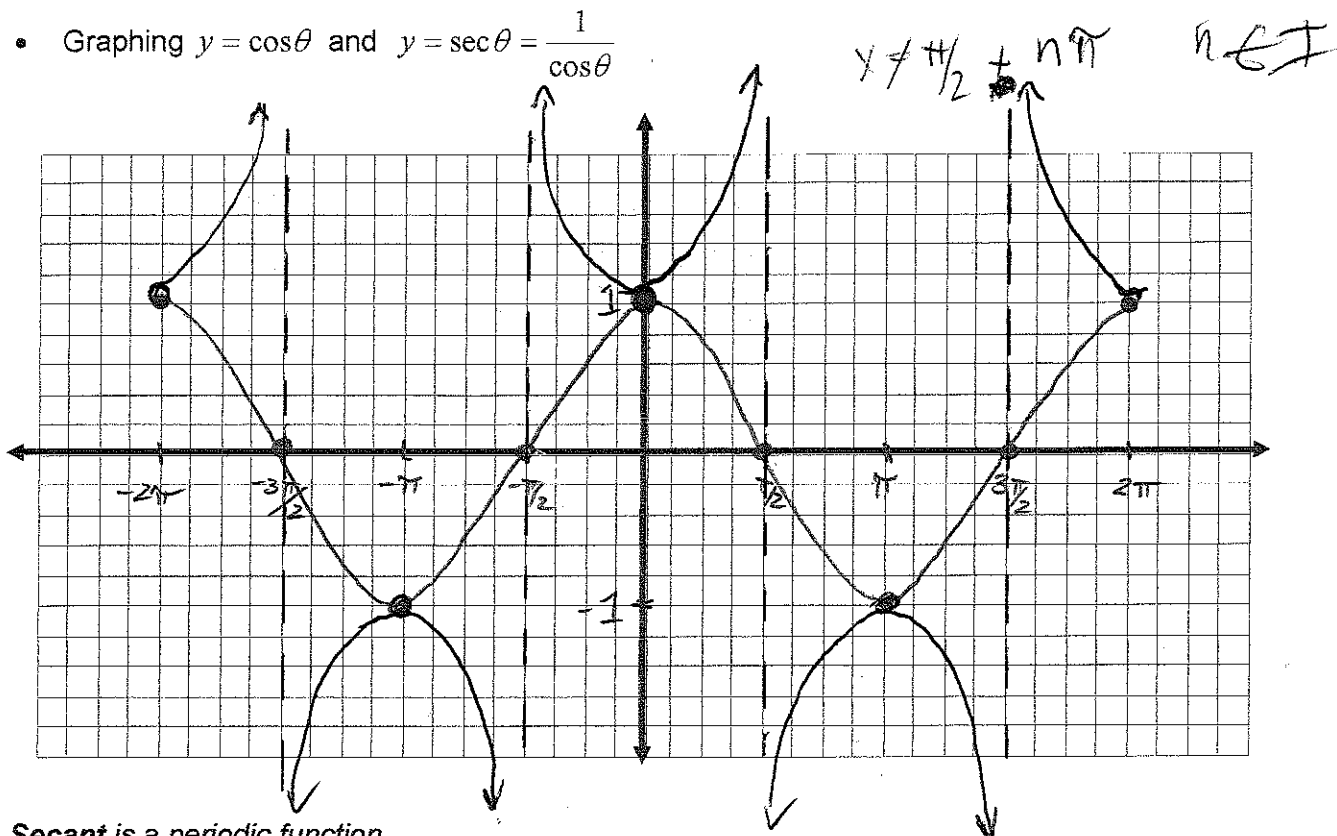- Graphing $y = \sin\theta$ and $y = \csc\theta = \dfrac{1}{\sin\theta}$

***Cosecant** is a periodic function*

1) Vertical asymptotes at: ____ $X = \pm n\pi$ where n is an integer. ____

2) Has the same period as: ____ $y = \sin\theta$, period $= 2\pi$ or $360°$ ____

3) Has the domain: ____ D: $\{ X \mid x \in \mathbb{R}, x \neq \pm n\pi$ where n is an integer.$\}$ ____

4) Has the range: ____ R: $\{ y \mid y \in \mathbb{R}, y > 1, y < -1 \}$. ____

- Graphing $y = \cos\theta$ and $y = \sec\theta = \dfrac{1}{\cos\theta}$

$x \neq \frac{\pi}{2} \pm n\pi \qquad n \in \mathbb{I}$

Secant is a periodic function

1) Vertical asymptotes at: $X = \pm n^{\pi}/_2$ where n is an integer $1, 3, 5, 7, etc \ldots$,

2) Has the same period as: $y = \cos\theta$, period $= 2\pi$

3) Has the domain: $\{ x \mid x \in \mathbb{R}, x \neq \pm n^{\pi}/_2$, where n is an integer $1, 3, 5, 7 etc \ldots \}$

4) Has the range: $\{ y \mid y \in \mathbb{R}, y \geq 1, y \leq -1 \}$.
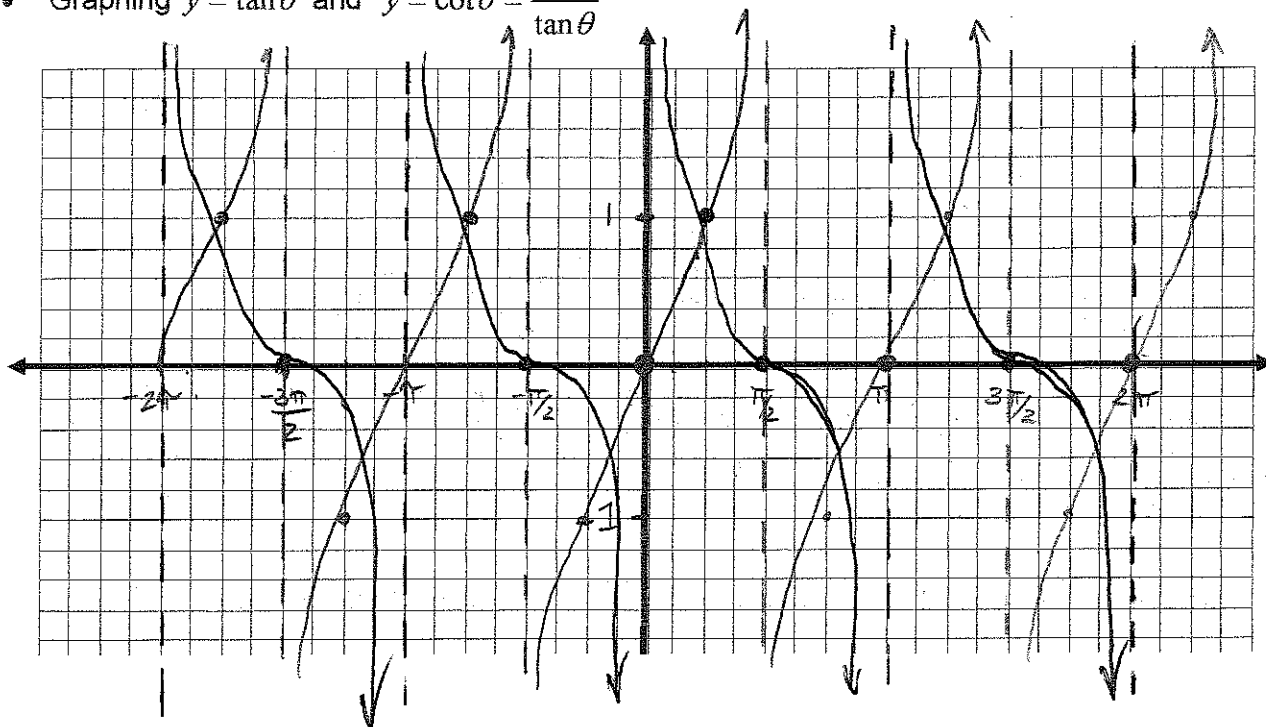
- Graphing $y = \tan\theta$ and $y = \cot\theta = \dfrac{1}{\tan\theta}$

Cotangent is a periodic function

1) Vertical asymptotes at: $X = \pm n\pi$ where n is an integer.

2) Has the same period as: $y = \tan\theta$, period $= 180°$

3) Has the domain: $\{x \mid x \in \mathbb{R}, x \neq$

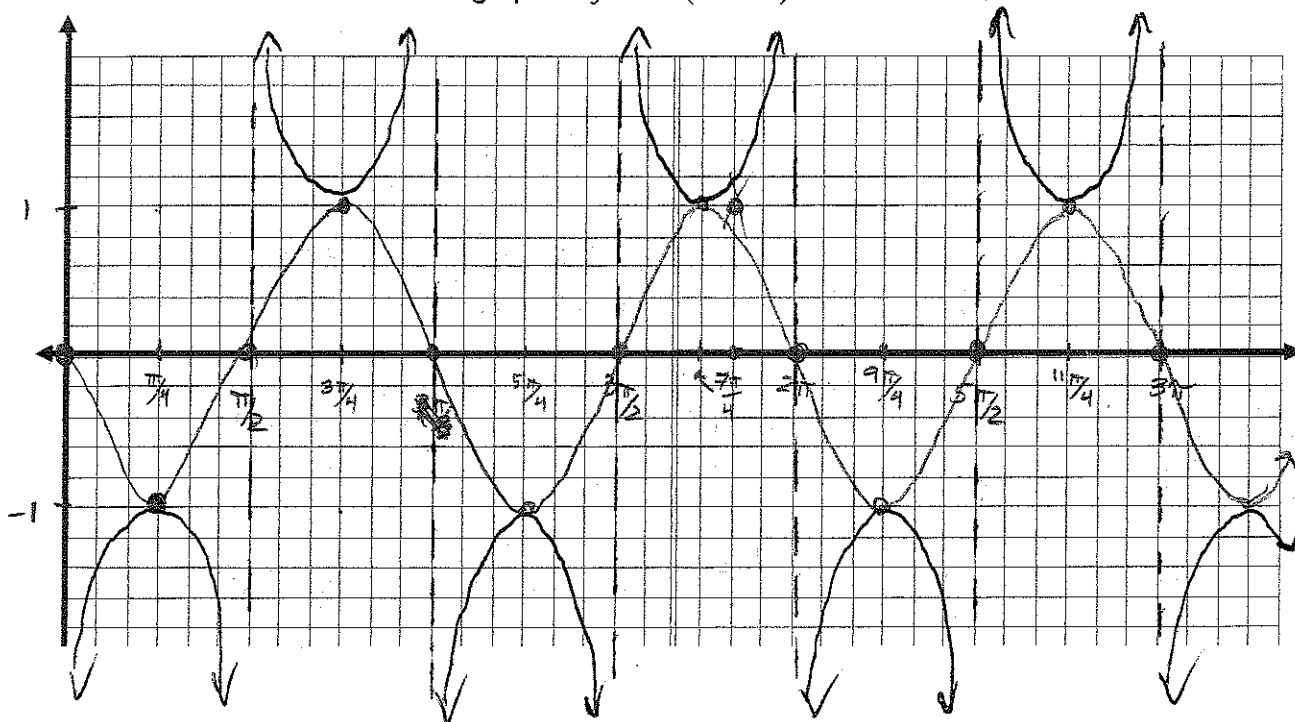4) Has the range: $\{y \mid y \in \mathbb{R}\}$

Summary:

The graph of a reciprocal function is related to the graph of its primary trigonometric function in the following ways:

1. The graph of the reciprocal function has a **vertical asymptote** at each zero of the corresponding primary trigonometric function.

2. The reciprocal function has the **same positive/negative** intervals as the corresponding primary trigonometric function.

3. **Intervals of increase** on the reciprocal function are **intervals of decrease** on the corresponding primary trigonometric function.

4. A reciprocal function intersects its primary function at points where the y-coordinate is **1** or **-1.**

5. If the primary trigonometric function has a local **minimum point**, the corresponding reciprocal function has a local **maximum point** at the same x-value. If the primary trigonometric function has a local **maximum point**, the corresponding reciprocal function has a local **minimum point** at the same x-value.

Example

Use transformations to sketch the graph of $y = \csc(2x + \pi)$

1) graph function: $y = \sin 2(x + \frac{\pi}{2})$
   or
   $y = \sin 2x$

period $= \frac{2\pi}{k}$
$= \frac{2\pi}{2}$
$= \pi$