Name: ______________________________ Date: ______________

Graphing and Modelling with Trigonometric Functions

⇒ Many phenomena in real-life can be modeling using a sine function or a cosine function. For example, sound waves are sinusoidal and can be modeled using a certain transformation of the function y = sinx

⇒ In order to model a real-world situation using a sine or a cosine function, you must analyze the situation and then transform the amplitude, period, vertical shift, and phase shift accordingly

Example 1

(a) Describe the transformations that must be applied to the graph of $f(x) = \sin x$ to obtain the graph of $g(x) = 4\sin 3x + 1$. Apply these transformations to sketch the graph of g(x).

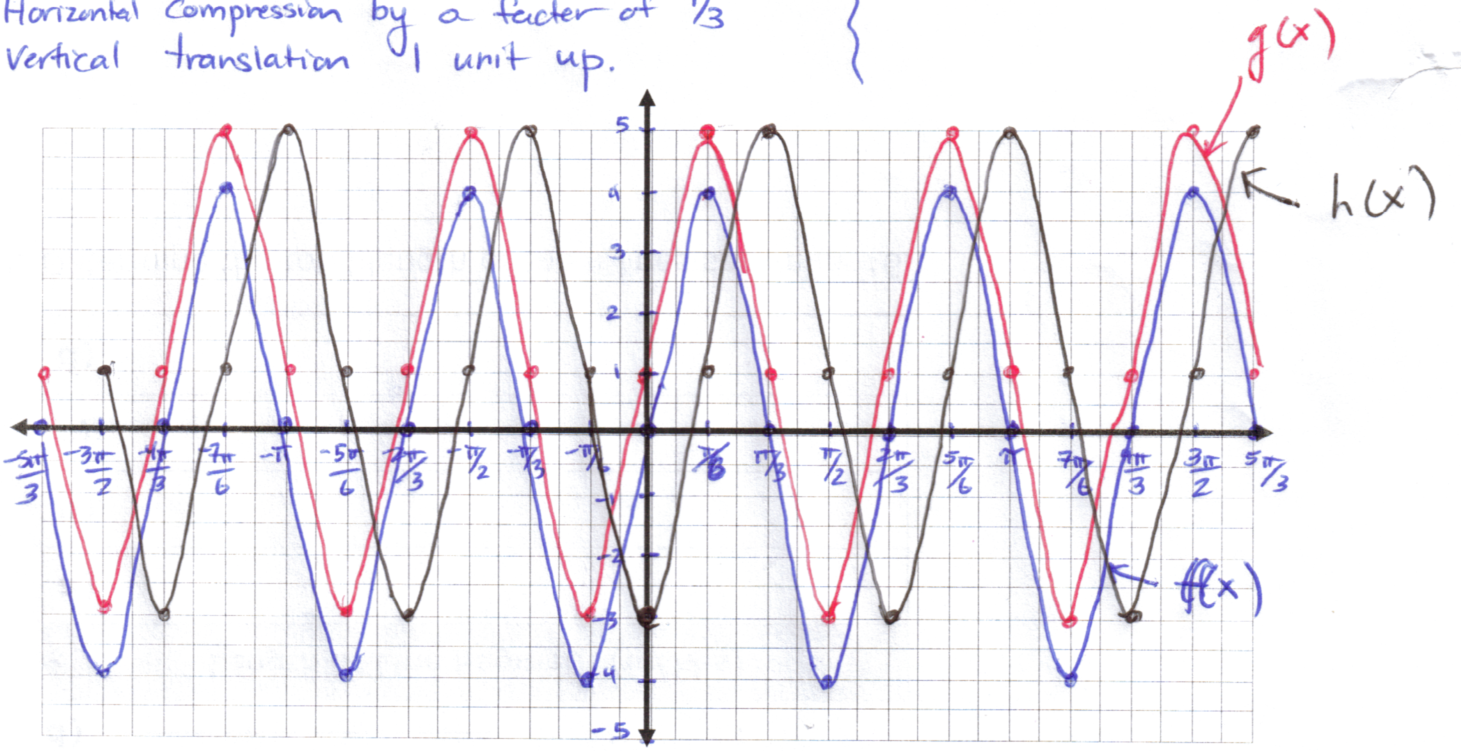
if $f(x) = \sin x$,

transformation to $f(x)$ to obtain $g(x)$ are:

- Vertical stretch by a factor of 4
- Horizontal Compression by a factor of 1/3
- Vertical translation 1 unit up.

$g(x) = 4\sin 3x + 1$

$\text{period} = \frac{2\pi}{3} = 120°$



(b) Modify the equation for $g(x)$ to include a phase shift of 30° to the right. Call this function $h(x)$. Apply the phase shift to the graph of $g(x)$ and transform it to $h(x)$.

$$h(x) = 4\sin\left[3\left(x - \frac{\pi}{6}\right)\right] + 1$$

Name: ______________________________ Date: ______________

Example 2

A sinusoidal function has an amplitude of 3 units, a period of $180°$ and a maximum at (0, 5). Represent the function with an equation in two different ways.

$\pi = \frac{2\pi}{k}$

$\frac{\cancel{\pi} k}{\cancel{\pi}} = \frac{2\cancel{\pi}}{\cancel{\pi}}$

$k = 2$

$A = \frac{max - min}{2}$

$3 = \frac{5 - x}{2}$

$6 = 5 - x$

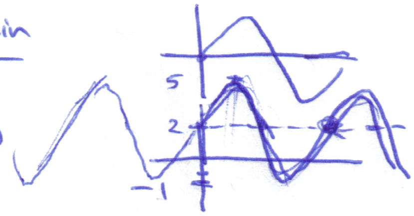
$1 = -x$

$-1 = x$

$M.V = \frac{Max + Min}{2}$

$= \frac{5 + (-1)}{2}$

$M.V = 2$



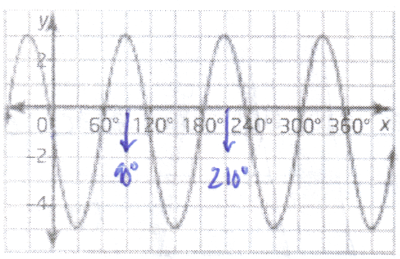
$y = 3 \sin 2x + 2$

or

$y = 3 \cos 2(x - \pi/4) + 2$

Example 3

Determine the equation of a sinusoidal function that represents each graph.



$A = \frac{3 - (-5)}{2}$

$A = 4$

$M.V = \frac{3 + (-5)}{2}$

$= -1$

Period $= 210° - 90° = 120°$

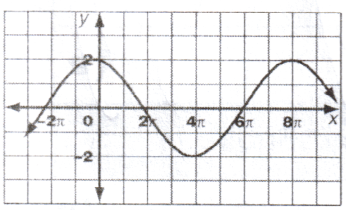
$\therefore$ period $= \frac{2\pi}{3}$

$\frac{2\pi}{3} = \frac{2\pi}{k}$

$2\pi k = 6\pi$

$k = 6\pi/2\pi \quad k = 3$

$\therefore y = -4\sin[3x] - 1$



$A = 2$

$M.V = 0$

period $= 8\pi$

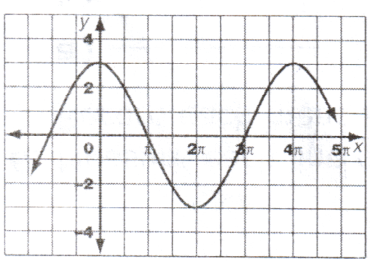
$8\pi = \frac{2\pi}{k}$

$8\pi k = 2\pi$

$k = 2\pi/8\pi$

$k = 1/4$

$\therefore y = 2 \cos \frac{1}{4} x$



$A = 3$

$M.V = \emptyset$

period $= 4\pi$

$4\pi = \frac{2\pi}{k}$

$\frac{\cancel{4\pi} k}{\cancel{4\pi}} = \frac{2\pi}{4\pi}$

$k = 1/2.$

$\therefore y = 3 \cos \frac{1}{2} x$

or

$y = -3 \sin[\frac{1}{2}(x - \pi)].$

Name: ______________________________ Date: ____________

Example 4

The height, h, in metres, above the ground of a rider on a Ferris wheel after t seconds can be modeled by the sine function $h(t) = 10\sin[3(t-30)] + 12$

Determine each of the following:

(a) The maximum and minimum heights of the rider above the ground
(b) The height of the rider above the ground after 30s
(c) The time required for the Ferris wheel to complete one revolution

(a) M.V = 12, max = 12 + 10 = 22
min = 12 − 10 = 2

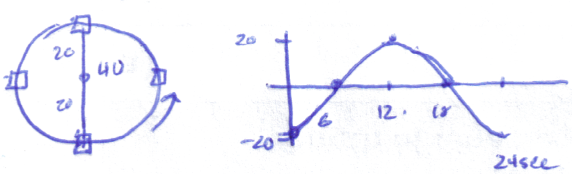
(b) $h(30) = 10\sin[3(30-30)] + 12$
$= 10\sin[3(0)] + 12$
$= 12$

(c) period $= \frac{360}{k}$
$= \frac{360}{3}$
$= 120$ seconds

Example 5

A ferris wheel has a diameter of 40 m and rotates once every 24 s.

(a) Draw a graph to show a person's height above or below the centre of rotation starting at the lowest position.
(b) Find an equation of the graph in (a).



$24 = \frac{360}{k}$
$24k = 360$
$k = 15$

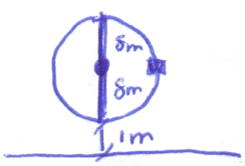
(b) $y = -20\cos 15t$

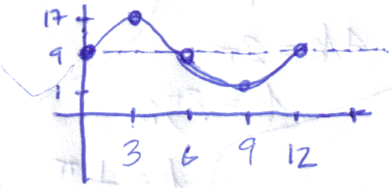
Name: ______________________________ Date: ______________

Example 6

A ferris wheel has a radius of 8 m and rotates once every 12 s. The bottom of the wheel is 1 m above the ground.

(a) Draw a graph to show a person's height above the ground starting at a position level with the centre.
(b) Find an equation of the graph in (a)





(b) $y = 8\sin\left[\frac{\pi}{6}t\right] + 9$

$A = \frac{17-1}{2}$

$= 8$

$M.V = \frac{17+1}{2}$

$= 9$

$12 = \frac{2\pi}{k}$

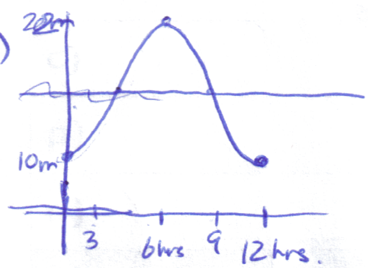
$\frac{12k}{12} = \frac{2\pi}{12}$ $k = \frac{\pi}{6}$

Example 7

During high tide, the water depth in a harbour is 22 m, and during low tide it is 10 m. (Assume a 12 h cycle)

(a) Find an expression for water depth t hours after low tide.
(b) Draw a graph of the function for a 48 h period.
(c) State the times at which water level is at (i) maximum (ii) minimum (iii) mean sea-level

a)



period = 12hrs.

$\therefore 12 = \frac{2\pi}{k}$

$12k = 2\pi$

$k = \frac{2\pi}{12}$

$k = \frac{\pi}{6}$

(a)

$y = -6\cos\frac{\pi}{6}t + 16$

or

$y = 6\sin\left[\frac{\pi}{6}(t-3)\right] + 16$

$A = \frac{22-10}{2}$

$= \frac{12}{2}$

$= 6$

$M.V = \frac{22+10}{2}$

$= 16$

(c) max : $t = 6, 18, 30, 42$ hours.

min : $t = 0, 12, 24, 36, 48$ hours

mean sea-Level : $t = 3, 9, 15, 21, 27, 33, 39, 45$ hours.

(b)