6.4: Power Law of Logarithms

Note:

- Exponenal equaons are equaons in which the exponent is the unknown variable.
- Because of the inverse relaonship between exponenal funcons and logarithmic funcons, logarithms provide a way to solve such equaons
- In order to solve exponenal equaons, we need a way of dealing with the unknown variable exponent

Recall:

- In secon 6.2 we aempted to find an approximate value for $\log_2 10$
- We did so by converng the logarithmic expression to an exponenal equaon and then using guess and check:

$$y = \log_2 10 \longrightarrow 2^y = 10$$

- Most scienfic calculators can only evaluate logarithms in base 10, so we need a new method to evaluate logarithms that are not base 10.

Power Law of Logarithms

Proving the Power Law:

- Let $w = \log_b x$

$$W = \log_b X$$

- Write the equaon in exponenal form:

$$b^W = X$$

- Raise both sides to the exponent *n:*

$$(b^w)^n = X^n$$

- Apply the power law of exponents

$$b^{wn} = X^n$$

- Rewrite in ~~exponenal~~ log form:

$$\log_b X^n = wn$$

- Substute $w = \log_b x$ to eliminate w:

$$\log_b X^n = (\log_b X)n$$

Power Law of Logarithms:

$$\log_b x^n = n\log_b x \qquad b > 0, b \neq 1, x > 0, n \in \mathbb{R}$$

Example 1: Apply the Power Law of Logarithms

Evaluate

(a) $\log_3 9^4$

(b) $\log_2 8^5$

(c) $\log 0.001^2$

(d) $\log_5 \sqrt{125}$

Sol'n:

a) $\log_3 9^4$

$y = \log_3 (3^2)^4$

$3^y = 3^8$

$\therefore y = 8$

$\log_3 9^4$

$4\log_3 9$

$4\log_3 3^2$

$= 4(2)$

$= 8$

$y = \log_3 3^2$

$3^y = 3^2$

(b) $\log_2 8^5$

$y = \log_2 8^5$

$2^y = 8^5$

$2^y = (2^3)^5$

$2^y = 2^{15}$

$\therefore y = 15$

(b) $\log_2 8^5$

$5\log_2 8$

$5\log_2 2^3$

$(5)(3)$

15

(c) $\log 0.001^2$

$10^y = 0.001^2$

$10^y = (10^{-3})^2$

$10^y = 10^{-6}$

$\therefore y = -6$

$\log 0.001^2$

$2\log 0.001$

$2\log 10^{-3}$

$2(-3)$

$= -6$

(d) $\log_5 \sqrt{125}$

$5^y = \sqrt{125}$

$5^y = 125^{1/2}$

$5^y = (5^3)^{1/2}$

$\therefore y = \frac{3}{2}$

$\log_5 \sqrt{125}$

$\log_5 (125)^{1/2}$

$\frac{1}{2} \log_5 5^3$

$\frac{1}{2}(3)$

$= 3/2$

Example 2:

Suppose you invest \$100 in an account that pays 5% interest, compounded annually. The amount, *A*, in dollars, in the account aer any given me, *t*, in years, is given by $A = 100(1.05)^t$. Find the me it takes for the inial amount of \$100 to double.

Sol'n:

$$A = 100(1.05)^t$$

$$\frac{200}{100} = \frac{\cancel{100}(1.05)^t}{\cancel{100}}$$

$$2 = (1.05)^t$$

$$\log 2 = \log (1.05)^t$$

★ take the common logarithm of both sides.

$$\frac{\log 2}{\log(1.05)} = \frac{t\,\cancel{\log(1.05)}}{\cancel{\log(1.05)}}$$

$$14.2 \doteq t$$

$\therefore$ It will take approx 14.2 years for \$100 to double.

Change of Base Formula

A formula can be used to determine the value of logarithms of any base

$b > 0, b \neq 1$

Proving the Change of Base Formula

- Let $w = \log_b m$

$$w = \log_b m$$

- Write the equaon in exponenal form:

$$b^w = m$$

- Take the common logarithm of both sides:

$$\log b^w = \log m$$

- Apply the power law of logarithms

$$w \log b = \log m$$

- Express in terms of ~~x~~ w:

$$w = \frac{\log m}{\log b}$$

Change of Base Formula:

Use this formula to calculate a logarithm with any base, by expressing it in terms of a common logarithm

$$\log_b m = \frac{\log m}{\log b} \qquad m > 0, b > 0, b \neq 1$$

Example 3: Evaluate Logarithms With Various Bases

Evaluate, correct to three decimal places

(a) $\log_5 17$

(b) $\log_{\frac{1}{2}} 10$

Sol'n:

(a) $\log_5 17 = \dfrac{\log 17}{\log 5} \doteq 1.760$

(b) $\log_{\frac{1}{2}} 10 = \dfrac{\log 10}{\log \frac{1}{2}} \doteq -3.322$

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