6.2: Logarithms

Recall:

$y = b^x \; ; \; x = b^y$

- Exponenal Funcons of the form: $y = b^x$ allow us to determine a value of y for each given value of x. Y is a funcon of x.
- Each x-value in the inverse graph gives a unique y-value, therefore the inverse is a funcon of x
- In the inverse funcon, the y-value is the exponent to which the base, b, must be raised to produce x, $b^y = x$
- In funcons, we prefer to write the y variable in terms of x, therefore the inverse relaonship is wrien as $\boxed{y = \log_b x}$

$b^y = x \qquad\qquad y = \log_b x$

Exponenal relaonships can be wrien using logarithm notaon:

$2^3 = 8 \longrightarrow 3 = \log_2 8$

$5^2 = 25 \longrightarrow 2 = \log_5 25$

$r^s = t \longrightarrow s = \log_r t$

The Logarithm Funcon

The **logarithm funcon** is defined as $y = \log_b x$ or *y* equals the logarithm of *x* to the base *b.*

This funcon is defined only for $b > 0, b \neq 1$

Example 1

Re-write each equaon in logarithmic form

(a) $16 = 4^2$ $2 = \log_4 16$

(b) $3^{-3} = \frac{1}{27}$ $-3 = \log_3 \frac{1}{27}$

(c) $m = n^3$ $3 = \log_n m$

Example 2

Re-write each of the following in the exponenal form

(a) $2 = \log_5 25$ $\longrightarrow$ $5^2 = 25$

(b) $3 = \log_{10} 1000$ $\longrightarrow$ $10^3 = 1000$

(c) $y = \log_{\frac{1}{4}} x$ $\longrightarrow$ $\left(\frac{1}{4}\right)^y = x$

Note:

A **common logarithm** is a logarithm with a base of 10. It is not necessary to write the base for a common logarithm: $\log x = \log_{10} x$

Example 3

Evaluate each of the following:

(a) $\log_4 64$

(b) $\log_5 1$

(c) $\log_{10} 0.1$

(d) $\log_3 \sqrt{3}$

(e) $\log_{16} 4$

(f) $\log_2 \left(\frac{1}{8}\right)$

Sol'n:

(a) let $x = \log_4 64$

$4^x = 64$

$x = 3$

(b) let $x = \log_5 1$

$5^x = 1$

$\therefore x = 0$

(c) let $x = \log_{10} 0.1$

$10^x = 0.1$

$x = -1$

(d) let $x = \log_3 \sqrt{3}$

$3^x = \sqrt{3}$

$\therefore x = \frac{1}{2}$

(e) let $x = \log_{16} 4$

$16^x = 4$

$(4^2)^x = 4$

$4^{2x} = 4^1$

$2x = 1$

$x = 1/2$

(f) Let $x = \log_2 \left(\frac{1}{8}\right)$

$2^x = \frac{1}{8}$

$2^x = 8^{-1}$

$2^x = (2^3)^{-1}$

$2^x = 2^{-3}$

$x = -3$

Example 4: Approximate Logarithms

Find an approximate value for each logarithm

(a) $\log_2 10$ (b) $\log 2500$

Sol'n:

a) Let $y = \log_2 10$

$2^y = 10$

$2^3 = 8$

$2^4 = 16$

try: 3.5

$2^{3.5} \doteq 11.3$

try: 3.3

$2^{3.3} \doteq 9.8$

$2^{3.32} \doteq 9.98$

$\therefore y \doteq 3.32$

(b) $\log 2500 \doteq 3.40$