6.1: The Exponenal Funcon and Its Inverse

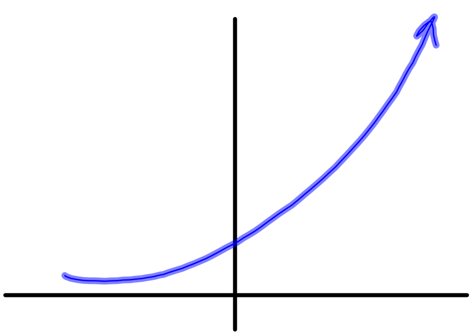
- Exponenal funcons are useful for describing relaonships
- If the growth of a populaon is proporonal to the size of the populaon as it grows, we describe the growth as exponenal

Recall:

- Exponenal Funcons of the form: $y = b^x$

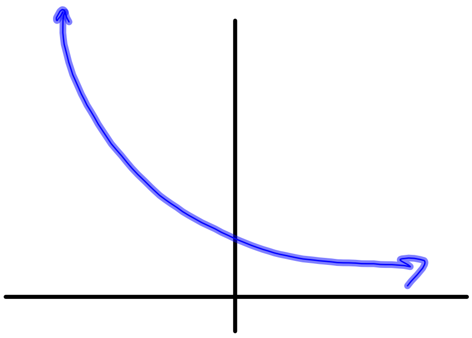
$b > 1$

Exponenal Growth



$0 < b < 1$

Exponenal Decay



Properties

- repeating pattern of finite differences.
- a rate of change that is increasing proportional to the function for $b > 1$.
- a rate of change that is decreasing proportional to the function for $0 < b < 1$.

Features that are the same:

1) both bases are positive.
2) both have the same y-int and it is +1.
3) both have the same horizontal asymptote at $y = 0$.
4) both have the same domain and it is $x \in \mathbb{R}$
5) both have the same range and it is $\{y | y > 0, y \in \mathbb{R}$
6) The graph is either increasing or decreasing.

Invesgate: The nature of the rate of change of an exponenal funcon

- Complete the table of values for: $y = 3^x$

x	y	Differences		
		$\Delta_1 y$	$\Delta_2 y$	$\Delta_3 y$
0	1			
1	3	2		
2	9	6	4	
3	27	18	12	8
4	81	54	36	24
5	243	162	108	72
6	729	486	324	216

for $y = 3^x$

Interval		Average Rate of Change, m_{AB}	Instantaneous Rate of Change at A, m_A	Instantaneous Rate of Change at B, m_B
A	B			
x=0	x=1	2	1.09	3.29
x=1	x=2	6	3.29	9.88
x=2	x=3	18	9.88	29.66
x=3	x=4	54	29.66	88.98
x=4	x=5	162	88.98	266.96

Example 1: Write an Equaon to Fit Data

Write an equaon to fit the data in the table of values

$y = ab^x$

x	y	$\Delta_1 y$
0	1	
1	4	3
2	16	12
3	64	48

×4 ×4 ×4

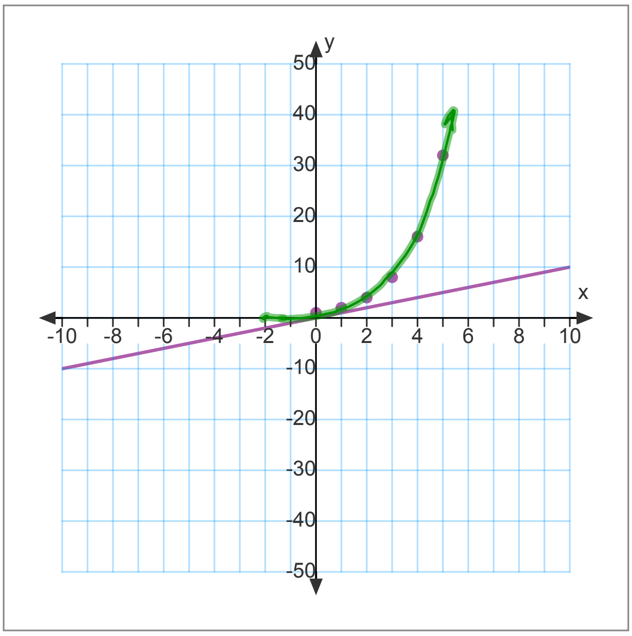
×4 ×4

$\therefore y = 4^x$

Invesgate: What is the nature of the inverse of an exponenal funcon?

$y = 2^x$

X	Y
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128



$y = x$

Example 2: Graphing an Inverse Funcon

Consider the funcon $f(x) = 4^x$

a) Idenfy the key features of the funcon

(i) domain and range

(ii) x-intercept, if it exists

(iii) y-intercept, if it exists

(iv) posve and negave intervals

(v) increasing and decreasing intervals

(vi) equaon of asymptote

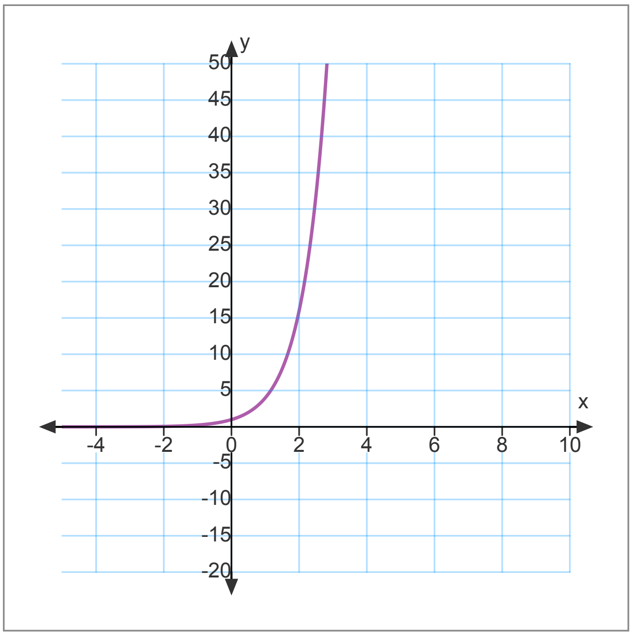
b) Sketch a graph of the funcon

c) On the same set of axes, sketch a graph of the inverse of the funcon

d) Idenfy the key features of the inverse of the funcon

$$y = 4^x$$

X	Y
0	1
1	4
2	8
3	32
4	128
5	512
6	2048
7	8192



key features:

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y > 0, y \in \mathbb{R}\}$

x-int: does not exist

y-int: +1.

Pos/neg intervals:
positive: $(-\infty, \infty)$

Increasing/dec. intervals
Increasing $(-\infty, \infty)$

Asymptote: $y = 0$

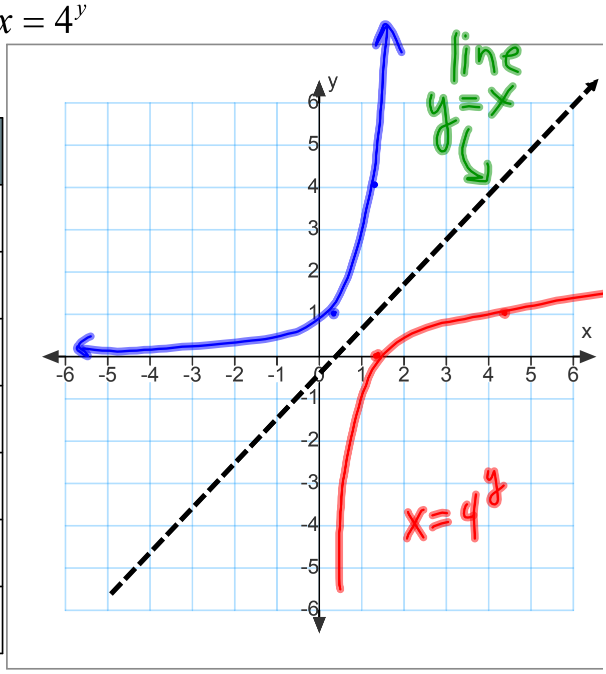
Table for: $y=4^x$

X	Y
0	1
1	4
2	8
3	32
4	128
5	512
6	2048
7	8192

Table for $x=4^y$

X	Y
1	0
4	1
8	2
32	3
128	4
512	5
2048	6

$x=4^y$



key features: of $x=b^y$

Domain: $\{x \mid x>0,\ x \in \mathbb{R}\}$

Range: $\{y \mid y \in \mathbb{R}\}$

x-int: +1

y-int: no y-int!

asymptote: $y=0$

Intervals increase/decrease

Increasing $(0,\infty)$

pos/neg intervals:

pos: $(0,1]$

neg: $[1,\infty)$

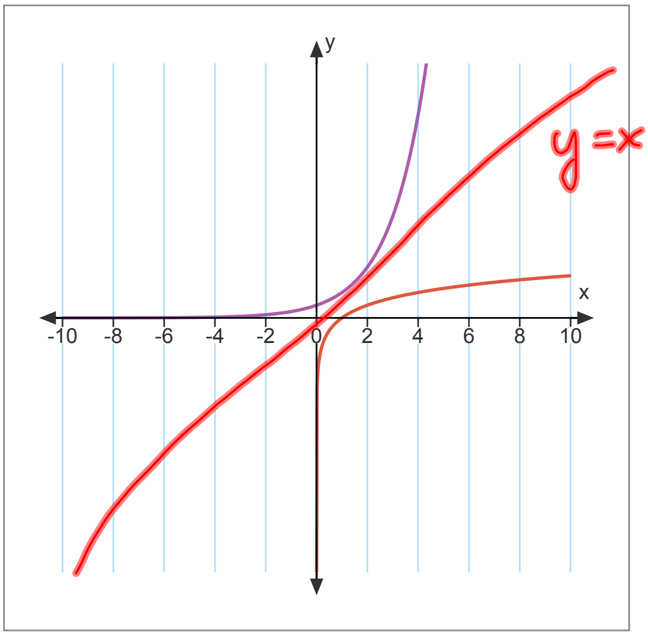
$y = 2^x$

$y = \log_2 x$

$y = b^x$

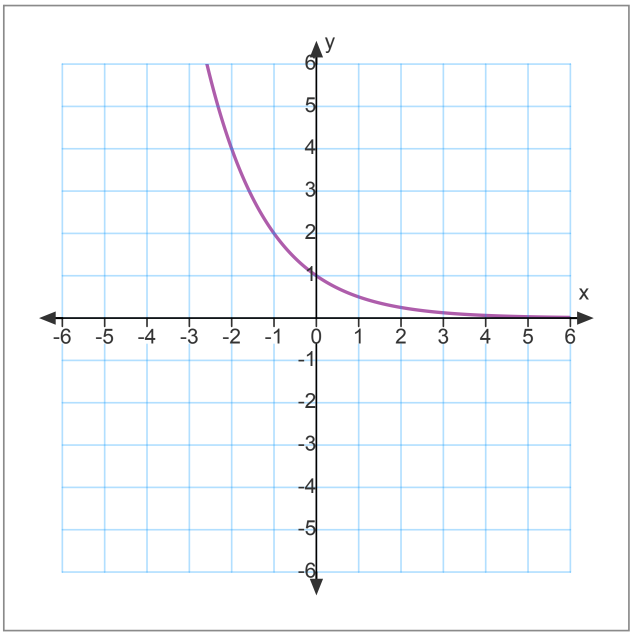
$x = b^y$

$y = \log_b x$



$$y = \left(\frac{1}{2}\right)^x$$

$$y = \log_{\frac{1}{2}} x$$



Properties of $x = b^y$

- The base, b, is always positive
- Vertical asymptote is $y=0$
- x-int is $+1$
- It is a reflection of $y=b^x$ about the line $y=x$
- For $b>1$, it is an increasing function.
- For $0<b<1$, it is a decreasing function
- Domain: $\{x \mid x>0, x \in \mathbb{R}\}$
- Range: $\{y \mid y \in \mathbb{R}\}$