5.5: Making Connecons and Instantaneous Rates of Change

Note:

- Sinusoidal models apply to many real-world phenomena that do not necessarily involve angles.
- The average and instantaneous rate of change of a sinusoidal funcon can be determined using the same strategies that were used for other types of funcons.

Recall:

For a funcon, *f(x),* the average rate of change in the interval $x_1 \le x \le x_2$

is $\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

Example 1

The posion of a parcle as it moves horizontally is described by the given equaon $s(t) = 12\sin\frac{\pi t}{90} + 15$ If *s* is the displacement, in metres, and *t* is the me, in seconds:

(a) Determine the average rate of change of *s(t)* in the following me intervals, rounded to three decimal places.

(i) 5 s to 10 s

$$\frac{\Delta s}{\Delta t} = \frac{s(10) - s(5)}{10 - 5} = \frac{12\sin\left(\frac{10\pi}{90}\right) + 15 - \left[12\sin\left(\frac{5\pi}{90}\right) + 15\right]}{10 - 5}$$

$$= \frac{19.10424172 - 17.08377813}{5}$$

$$= 0.404 \text{ m/s}$$

(ii) 9 s to 10 s

$$\frac{\Delta s}{\Delta t} = \frac{12\sin\left(\frac{10\pi}{90}\right) + 15 - \left[12\sin\left(\frac{9\pi}{90}\right) + 15\right]}{10 - 9}$$

$$= 0.396 \text{ m/s}$$

(iii) 9.9 s to 10 s

$$\frac{\Delta s}{\Delta t} = \frac{12\sin\left(\frac{10\pi}{90}\right) + 15 - \left[12\sin\left(\frac{9.9\pi}{90}\right) + 15\right]}{10 - 9.9}$$

$$= 0.394 \text{ m/s}$$

(b) Esmate a value for the instantaneous rate of change of *s(t)* at *t = 10 s*

approximately 0.39 m/s or 0.4 m/s

(c) What physical quanty does this instantaneous rate of change represent?

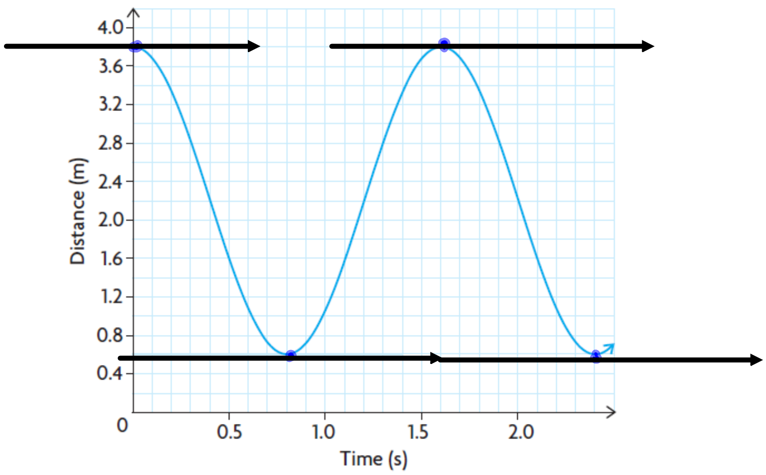
velocity or speed.

Example 2

Melissa used a moon detector to measure the horizontal distance between her and a child on a swing. She stood in front of the child and recorded the distance, *d(t)*, in metres, over a period of me, *t*, in seconds. The data she collected are given in the following tables and are shown on the graph below:

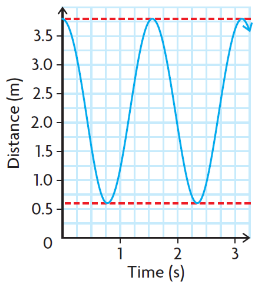
Time (s)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
Distance (m)	3.8	3.68	3.33	2.81	2.2	1.59	1.07	0.72	0.6	0.72	1.07	1.59

Time (s)	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4
Distance (m)	2.2	2.81	3.33	3.68	3.8	3.68	3.33	2.81	2.2	1.59	1.07	0.72	0.6



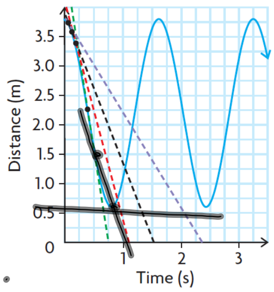
How did the speed of the child change as the child swung back and forth?

To determine how the child's speed changed as he/she swung back and forth we can analyze the data or we can analyze the graph by drawing tangent lines and seeing how their slope changes



When the child was at the farthest point and closest point from the moon detector, the instantaneous velocity was 0.

Between 0 s and about 0.4 s, the child's speed was increasing.

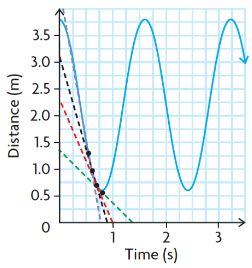


On this interval, the tangent lines become steeper as me increases.

$$speed = |velocity| = \left|\frac{\Delta dist}{\Delta time}\right|$$

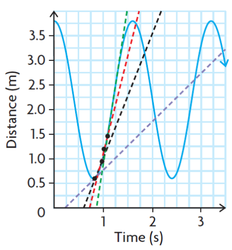
This means the magnitudes of the slopes are increasing. The tangent lines have negave slopes, which means the distance between the child and the moon detector connues to decrease.

Between 0.4 s and about 0.8 s, the child's speed was decreasing.



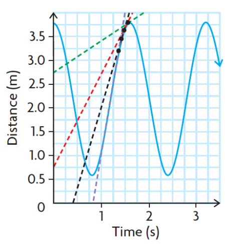
On this interval, the tangent lines are geng less steep as me increases. This means the magnitudes of the slopes are decreasing. The tangent lines sll have negave slopes, which means the distance between the child and the moon detector is sll decreasing. The child is slowing down as the swing approaches the point where a change in direcon occurs. The slopes indicate a change in the child's posion from toward the detector to away from the detector.

Between 0.8 s and about 1.2 s, the child's speed was increasing.



On this interval, the tangent lines are geng steeper as me increases. This means the magnitudes of the slopes are increasing. The tangent lines have posive slopes, which means the distance between the child and the moon detector is increasing. Therefore, the moon is away from the detector.

Between 1.2 s and about 1.6 s, the child's speed was decreasing.

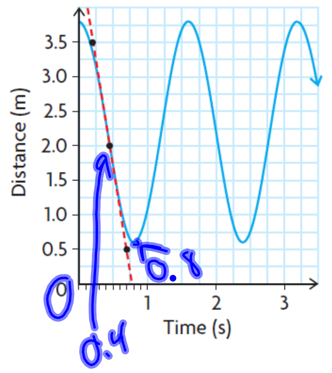


On this interval, the tangent lines are geng less steep as me increases. This means the magnitudes of the slopes are decreasing. The tangent lines sll have posive slopes, which means the distance between the child and the moon detector is sll increasing. The child is slowing down as the swing approaches the point where there is a change in direcon from away from the detector to toward the detector.

Example 3

To model the moon of the child on the swing, Melissa determined that she could use the equaon $d(t)=1.6\cos\left(\frac{\pi}{0.8}t\right)+2.2$ where *d(t)* is the distance from the child to the moon detector, in metres, and *t* is the me, in seconds. Use this equaon to esmate when the child was moving the fastest and what speed the child was moving at this me.

The child must have been moving the fastest at around 0.4 s. Drawing a tangent line at *t = 0.4* supports this, since the tangent line appears to be steepest here.



$h = 0.001$

$$d(t)=1.6\cos\left(\frac{\pi}{0.8}t\right)+2.2$$

To get a beer esmate of the child's speed at this me, use the difference quoent $\frac{d(a+h)-d(a)}{h}$ where *a = 0.4* Use a very small value for *h*.

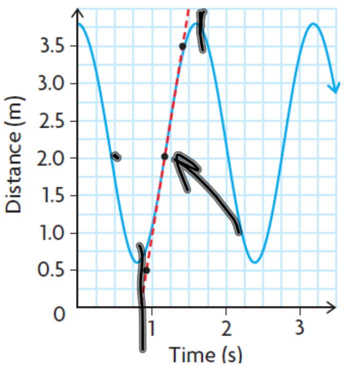
$$\frac{\Delta d}{\Delta t} = \frac{1.6\cos\left(\frac{\pi}{0.8}(0.4+h)\right)+2.2-\left[1.6\cos\left(\frac{\pi}{0.8}(0.4)\right)+2.2\right]}{0.001} \quad (h = 0.001)$$

$$\frac{\Delta d}{\Delta t} = \frac{1.6\cos\left(\frac{0.401\pi}{0.8}\right)+2.2-\left[1.6\cos\left(\frac{0.4\pi}{0.8}\right)+2.2\right]}{0.001}$$

$$= \frac{2.193716831 - 2.2}{0.001}$$

$$= -6.3 \text{ m/s}$$

The child was also travelling the fastest at around 1.2 s. Drawing a tangent line at $t = 1.2$ supports this, since the tangent line appears to be steepest here.



$h = 0.001$

$$d(t) = 1.6\cos\left(\frac{\pi}{0.8}t\right) + 2.2$$

To get a beer esmate of the child's speed at this me, use the difference quoent $\frac{d(a+h)-d(a)}{h}$ where $a = 1.2$ Use a very small value for h.

$$\frac{\Delta d}{\Delta t} = \frac{1.6\cos\left(\frac{\pi}{0.8}(1.2+0.001)\right) + 2.2 - \left[1.6\cos\left(\frac{1.2\pi}{0.8}\right) + 2.2\right]}{0.001}$$

$$= 6.3 \text{ m/s}$$

Consolidate:

The approximate value of the instantaneous rate of change can be determined using one of the stategies below:

- Sketching an approximate tangent line on the graph and esmang its slope using two points that lie on the secant line.
- Using two points in the table of values (preferably two points that lie on **either side** and/or **as close as possible** to the tangent point) to calculate the slope of the corresponding secant line.
- Using the defining equaon of the trigonometric funcon and a very small interval near the point of tangency to calculate the slope of the corresponding secant line.
- Using the difference quoent.