5.4: Solving Trigonometric Equaons

Note:

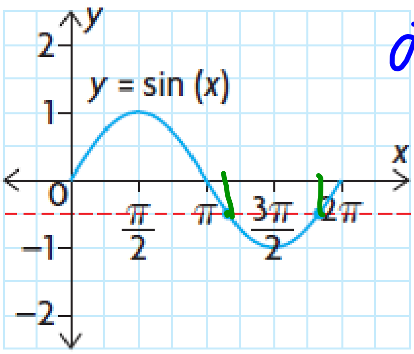
- A trigonometric equaon is an identy if the L.S = R.S for all values in the domain.
- Not all trigonometric equaons are idenes. If the equaon is only true for some values in the domain, then the equaon is not an identy.

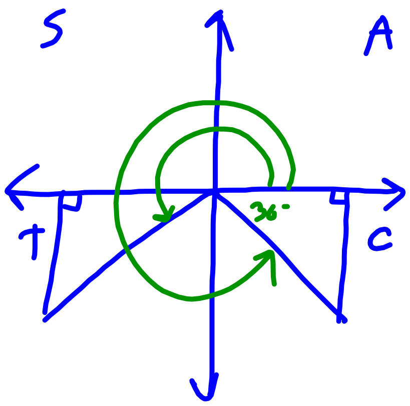
Example 1

Given the equaon $2\sin x + 1 = 0 \quad 0 \le x \le 2\pi$

a) Determine all soluons in the specified interval

b) Verify the soluons using a graph





a) $2\sin x + 1 = 0$

$2\sin x = -1$

$\sin x = \frac{-1}{2}$

$x = \sin^{-1}\left(\frac{-1}{2}\right) \quad x = -30°$

$180° + 30° = 210°$

$270° + 60° = 330°$

Example 2

Solve $3(\tan x + 1) = 2 \quad 0° \le x \le 360°$ correct to 1 decimal place.

$3(\tan x + 1) = 2$

$3\tan x + 3 = 2$

$3\tan x = -1$

$\tan x = -\frac{1}{3}$

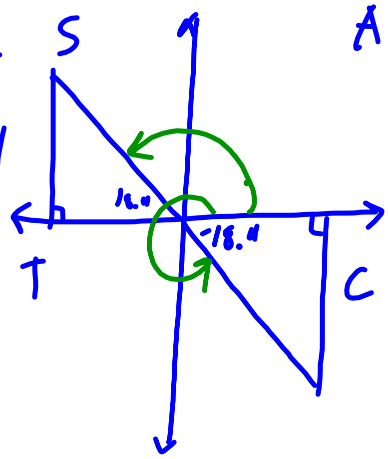
$\tan x + 1 = \frac{2}{3}$

$\tan x = \frac{2}{3} - 1$

$\tan x = \frac{-1}{3}$

$x = \tan^{-1}\left(\frac{-1}{3}\right)$

$x = -18.4$



Sol: $90° + 71.6° = 161.6°$

$270° + 71.6° = 341.6°$

Example 3

Solve $2\sin\theta\cos\theta=\cos 2\theta$ for θ in the interval $0\le\theta\le 2\pi$

$$2\sin\theta\cos\theta=\cos 2\theta$$

$$\frac{\sin 2\theta}{\cancel{\cos 2\theta}}=\frac{\cancel{\cos 2\theta}}{\cos 2\theta}$$

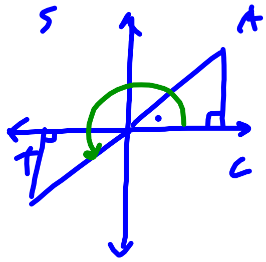
$$\tan 2\theta = 1$$

let x be 2θ

$\tan x = 1$

$x=\tan^{-1}(1)$

$x=\frac{\pi}{4}$



$x=\frac{5\pi}{4}$

$2\theta=\frac{\pi}{4}$ $\quad 2\theta=\frac{5\pi}{4}$

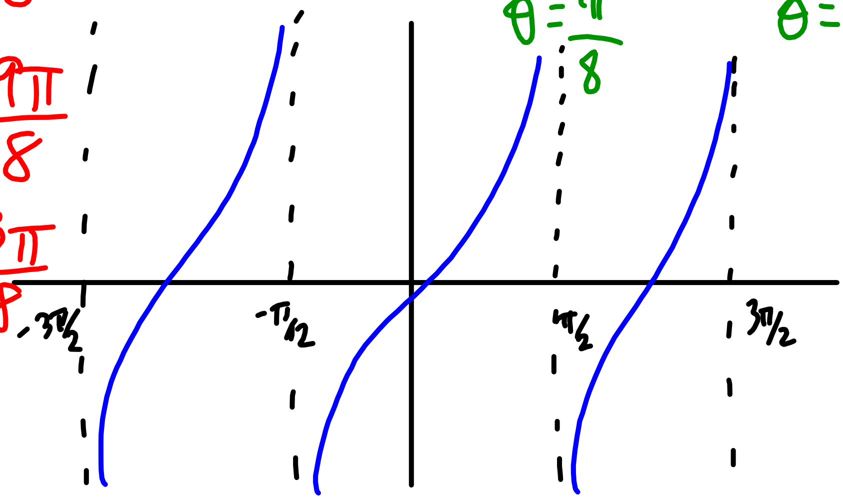
$\theta=\frac{\pi}{8}$ $\quad \theta=\frac{5\pi}{8}$

other 2 sol'ns:

$\frac{\pi}{8}+\frac{\pi}{2}=\frac{5\pi}{8}$

$\frac{5\pi}{8}+\frac{\pi}{2}=\frac{9\pi}{8}$

$\frac{9\pi}{8}+\frac{\pi}{2}=\frac{13\pi}{8}$



$y=\tan x$

$y=\tan 2x$

period $=\frac{\pi}{2}$

Example 4

Solve each equaon for x in the interval $0 \le x \le 2\pi$

(a) $\sin^2 x - \sin x = 2$

(b) $2\sin^2 x - 3\sin x + 1 = 0$

(a) $\sin^2 x - \sin x = 2$

$\sin^2 x - \sin x - 2 = 0$

let sinx be y

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

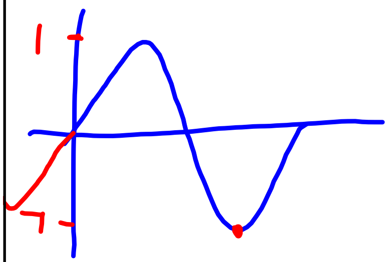
$(\sin x - 2)(\sin x + 1) = 0$

$\sin x - 2 = 0$ or $\sin x + 1 = 0$

$\sin x = 2$ $\qquad$ $\sin x = -1$

no sol'n !! :($\qquad$ $x = \sin^{-1}(-1)$

$x = -90°$



sol'n: $3\pi/2$

(b) $2\sin^2 x - 3\sin x + 1 = 0$ $\qquad$ $2x^2 - 3x + 1 = 0$

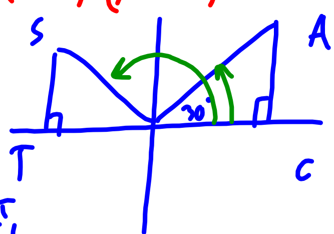
$(2\sin x - 1)(\sin x - 1) = 0$ $\qquad$ $2x^2 - 2x - 1x + 1 = 0$

$2\sin x - 1 = 0$ $\qquad$ $\sin x - 1 = 0$ $\qquad$ $2x(x-1) - 1(x-1) = 0$

$\sin x = \frac{1}{2}$ $\qquad$ $\sin x = 1$ $\qquad$ $(2x-1)(x-1) = 0$

$x = \sin^{-1}(1)$

$x = \pi/2$



$x = \sin^{-1}\left(\frac{1}{2}\right)$

$x = 30°$ or $\pi/6$

$x = 180° - 30°$

$x = 150°$

sol'n: $\pi/6$, $\pi/2$, $\frac{5\pi}{6}$

Example 5

For each equaon, use a trigonometric identy to create a quadrac equaon. Then solve the equaon for x in the interval $[0, 2\pi]$

(a) $2\sec^2 x - 3 + \tan x = 0$

(b) $2\sin x + 3\cos 2x = 2$

$\sec^2 x = 1 + \tan^2 x$

(a) $2\sec^2 x - 3 + \tan x = 0$

$2(1+\tan^2 x) - 3 + \tan x = 0$

$2 + 2\tan^2 x - 3 + \tan x = 0$

$2\tan^2 x + \tan x - 1 = 0$

$(\tan x + 1)(2\tan x - 1) = 0$

$2x^2 + x - 1 = 0$

$2x^2 + 2x - 1x - 1 = 0$

$2x(x+1) - 1(x+1) = 0$ $\times : -2$

$(x+1)(2x-1) = 0$ $+ : 1$

$\tan x + 1 = 0$

$\tan x = -1$

$x = \tan^{-1}(-1)$

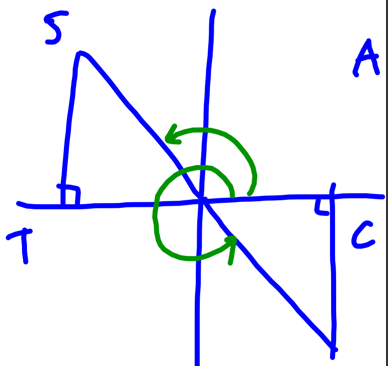
$x = \frac{-\pi}{4}$

$2\tan x - 1 = 0$

$\tan x = \frac{1}{2}$

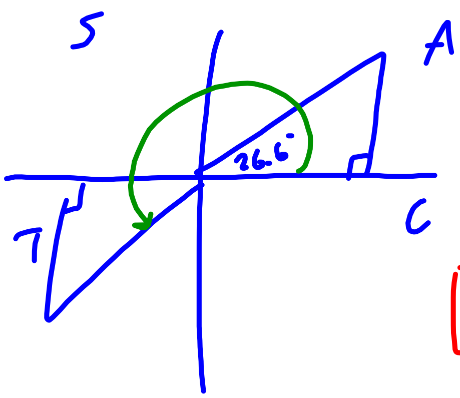
$x = \tan^{-1}\left(\frac{1}{2}\right)$

$x = 26.6^\circ$



$\boxed{x = 135^\circ}$

$\boxed{x = 315^\circ}$



$\boxed{x = 26.6^\circ}$

$x = 180^\circ + 26.6^\circ$

$\boxed{x = 206.6^\circ}$

$$2\sin x + 3\cos 2x = 2$$

$2\sin x + 3(1 - 2\sin^2 x) = 2$

$2\sin x + 3 - 6\sin^2 x - 2 = 0$

$-6\sin^2 x + 2\sin x + 1 = 0$

$6\sin^2 x - 2\sin x - 1 = 0$

* use quad formula to solve.

Need to Know

• Because of their periodic nature, trigonometric equaons have an infinite number of soluons. When we use a trigonometric model, we usually want soluons within a specified interval.

• To solve a linear trigonometric equaon, use special triangles, a calculator, a sketch of the graph, and/or the CAST rule.

• A scienfic or graphing calculator provides very accurate esmates of the value for an inverse trigonometric funcon. The inverse trigonometric funcon of a posive rao yields the related angle. Use the related acute angle and the period of the corresponding funcon to determine all the soluons in the given interval.

• You can use a graphing calculator to verify the soluons for a linear trigonometric equaon by

- graphing the appropriate funcons on the graphing calculator and determining the points of intersecon
- graphing an equivalent single funcon and determining its zeros