4.5 #10) Verify that $\sin\left(x-\frac{4\pi}{3}\right) = \frac{\sqrt{3}\cos x - \sin x}{2}$ by applying a double-angle formula.

recall: $\sin 2x = 2\sin x\cos x$ also $\cos 2x = \cos^2 x - \sin^2 x$

let $2y = \left(x - \frac{4\pi}{3}\right)$, $\therefore y = \frac{1}{2}\left(x-\frac{4\pi}{3}\right)$

$\therefore$ L.S $= \sin\left(x-\frac{4\pi}{3}\right)$

$= \sin 2y$

$= 2\sin y\cos y$

$$L.S = 2\sin\left[\frac{1}{2}\left(x-\frac{4\pi}{3}\right)\right]\cos\left[\frac{1}{2}\left(x-\frac{4\pi}{3}\right)\right]$$

$$= 2\sin\left[\left(\frac{x}{2}-\frac{2\pi}{3}\right)\right]\cos\left[\left(\frac{x}{2}-\frac{2\pi}{3}\right)\right]$$

$$L.S = 2\left[\sin\left(\tfrac{x}{2}\right)\cos\left(\tfrac{2\pi}{3}\right) - \cos\left(\tfrac{x}{2}\right)\sin\left(\tfrac{2\pi}{3}\right)\right]\left[\cos\left(\tfrac{x}{2}\right)\cos\left(\tfrac{2\pi}{3}\right) + \sin\left(\tfrac{x}{2}\right)\sin\left(\tfrac{2\pi}{3}\right)\right]$$

$$= 2\left[\sin(x/2)(-1/2) - \cos(x/2)\sqrt{3}/2\right]\left[\cos(x/2)(-1/2) + \sin(x/2)\sqrt{3}/2\right]$$

$$= 2\left[-\frac{\sin(x/2)}{2} - \frac{\sqrt{3}\cos(x/2)}{2}\right]\left[-\frac{\cos(x/2)}{2} + \frac{\sqrt{3}\sin(x/2)}{2}\right]$$

$$= 2\left[\frac{-\sqrt{3}\cos(x/2) - \sin(x/2)}{2}\right]\left[\frac{\sqrt{3}\sin(x/2) - \cos(x/2)}{2}\right]$$

$$= \frac{-3\cos(x/2)\sin(x/2) + \sqrt{3}\cos^2(x/2) - \sqrt{3}\sin^2(x/2) + \cos(x/2)\sin(x/2)}{2}$$

$$= \frac{-2\cos(x/2)\sin(x/2) + \sqrt{3}\left[\cos^2(x/2) - \sin^2(x/2)\right]}{2}$$

$$= \frac{-\sin \cancel{2}(x/\cancel{2}) + \sqrt{3}\left[\cos 2(x/2)\right]}{2}$$

$$= \frac{\sqrt{3}\cos x - \sin x}{2} \qquad \therefore L.S = R.S$$

4.5

#13 Apply double-angle formulas to verify the identity $\sin 4x = 2\sin 2x \cos 2x$

☆ recall: $\sin 2x = 2\sin x \cos x$

Let $y = 2x$

$\therefore$ ~~Sin~~ $L.S = \sin 4x$

$$
\begin{aligned}
R.S &= 2\sin 2x \cos 2x \\
&= 2\sin y \cos y. \\
&= \sin 2y. \\
&= \sin 2(2x) \\
&= \sin 4x
\end{aligned}
$$

$\therefore L.S = R.S.$

4.5

#17) Prove that $\frac{\sin 2x}{1-\cos 2x} = \cot x$

$$LS = \frac{\sin 2x}{1-\cos 2x}$$

$$= \frac{2\sin x \cos x}{1-[2\cos^2 x - 1]}$$

$$= \frac{2\sin x \cos x}{1-2\cos^2 x + 1}$$

$$= \frac{2\sin x \cos x}{2-2\cos^2 x}$$

$$= \frac{2\sin x \cos x}{2(1-\cos^2 x)}$$

$$= \frac{\cancel{2}\cancel{\sin x}\cos x}{\cancel{2}\sin^{\cancel{2}} x}$$

$$= \frac{\cos x}{\sin x}$$

$$= \cot x$$

L.S = R.S

4.5

#181 Prove that $\frac{2\csc 2x \tan x}{\sec x} = \sec x$

$$L.S = \frac{2\csc 2x \tan x}{\sec x}$$

$$= \frac{2\left(\frac{1}{\sin 2x}\right)\frac{\sin x}{\cos x}}{\frac{1}{\cos x}}$$

$$= \left(\frac{2\sin x}{\sin 2x \cos x}\right)\left(\frac{\cos x}{1}\right)$$

$$= \frac{2\sin x \cancel{\cos x}}{\sin 2x \cancel{\cos x}}$$

$$= \frac{2\sin x}{\sin 2x}$$

$$= \frac{\cancel{2\sin x}}{\cancel{2\sin x}\cos x}$$

$$= \frac{1}{\cos x}$$

$$= \sec x$$

$$L.S = R.S.$$

$$1 + \cos x = \frac{\sin^2 x}{1 - \cos x}$$

$L.S = 1 + \cos x$

$$R.S = \frac{\sin^2 x}{1 - \cos x}$$

$$= \frac{(1 - \cos^2 x)}{1 - \cos x}$$

$$= \frac{\cancel{(1 - \cos x)}(1 + \cos x)}{\cancel{(1 - \cos x)}}$$

$$= 1 + \cos x$$

Since $L.S = R.S$

Hence, $1 + \cos x = \dfrac{\sin^2 x}{1 - \cos x}$

Prove: $\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \sec 2\theta - \tan 2\theta$

$$R.S = \sec 2\theta - \tan 2\theta$$

$$= \frac{1}{\cos 2\theta} - \frac{\sin 2\theta}{\cos 2\theta}$$

$$= \frac{1 - \sin 2\theta}{\cos 2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{(\sin\theta - \cos\theta)(\sin\theta - \cos\theta)}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}$$

$$= \frac{(-\cos\theta + \sin\theta)(\sin\theta - \cos\theta)}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}$$

$$= \frac{-1\,\cancel{(\cos\theta - \sin\theta)}(\sin\theta - \cos\theta)}{\cancel{(\cos\theta - \sin\theta)}(\cos\theta + \sin\theta)}$$

$$= \frac{-1(-\cos\theta + \sin\theta)}{\cos\theta + \sin\theta}$$

$$= \frac{\cancel{-1}\,\cancel{(-1)}(\cos\theta - \sin\theta)}{\cos\theta + \sin\theta)}$$

$$= \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$L.S = R.S.$

Prove: $\dfrac{1+\cos 2x}{\sin 2x} = \cot x$

$$L.S = \frac{1+\cos 2x}{\sin 2x}$$

$$= \frac{\cancel{\sin^2 x} + \cos^2 x + \cos^2 x - \cancel{\sin^2 x}}{2\sin x \cos x}$$

$$= \frac{\cancel{2}\cos^{\cancel{2}} x}{\cancel{2}\sin x \cancel{\cos x}}$$

$$= \frac{\cos x}{\sin x}$$

$$= \cot x$$

$$L.S = R.S.$$

$$\frac{\cos(a-b)}{\cos(a+b)} = \frac{1+\tan a \tan b}{1-\tan a \tan b}$$

$$L.S = \frac{\cos a \cos b + \sin a \sin b}{\cos a \cos b - \sin a \sin b}$$

$$R.S = \frac{1+\left(\frac{\sin a}{\cos a}\right)\left(\frac{\sin b}{\cos b}\right)}{1-\left(\sin a/\cos a\right)\left(\sin b/\cos b\right)}$$

$$= \frac{1+\frac{\sin a \sin b}{\cos a \cos b}}{1-\frac{\sin a \sin b}{\cos a \cos b}}$$

$$=$$

$$= \frac{\frac{\cos a \cos b}{\cos a \cos b}+\frac{\sin a \sin b}{\cos a \cos b}}{\frac{\cos a \cos b}{\cos a \cos b}-\frac{\sin a \sin b}{\cos a \cos b}}$$

$$= \frac{\frac{\cos a \cos b + \sin a \sin b}{\cos a \cos b}}{\frac{\cos a \cos b - \sin a \sin b}{\cos a \cos b}}$$

$$= \left(\frac{\cos a \cos b + \sin a \sin b}{\cancel{\cos a \cos b}}\right)\left(\frac{\cancel{\cos a \cos b}}{\cos a \cos b - \sin a \sin b}\right)$$

$$= \frac{\cos a \cos b + \sin a \sin b}{\cos a \cos b - \sin a \sin b}$$

$$L.S = R.S.$$