

4.4  
#7

Find exact value.

$$a) \sin\left(\frac{23\pi}{12}\right) = ?$$

$$\sin\left(\frac{23\pi}{12}\right) = \sin\left(2\pi - \frac{\pi}{12}\right) \quad \star 345^\circ \text{ is broken up into the difference of } 360^\circ \text{ and } 15^\circ.$$

$$\sin\left(2\pi - \frac{\pi}{12}\right) = \sin(2\pi)\cos\left(\frac{\pi}{12}\right) - \cos(2\pi)\sin\left(\frac{\pi}{12}\right) \quad \star \text{ apply difference formula for sine.}$$

$$\sin\left(2\pi - \frac{\pi}{12}\right) = (0)\cos\left(\frac{\pi}{12}\right) - (1)\sin\left(\frac{\pi}{12}\right)$$

$$\sin\left(2\pi - \frac{\pi}{12}\right) = -\sin\left(\frac{\pi}{12}\right)$$

we need an exact value for the sine of  $15^\circ$

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) \quad \star 15^\circ \text{ is broken up into the difference of } 45^\circ \text{ and } 30^\circ.$$

$$\sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$\sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\therefore \sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\therefore \sin\left(\frac{23\pi}{12}\right) = -\sin\left(\frac{\pi}{12}\right)$$

$$\therefore \sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\therefore \sin\left(\frac{23\pi}{12}\right) = -\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$$

or  $\frac{1-\sqrt{3}}{2\sqrt{2}}$