

4.4 #7(c)

7

$$(c) \tan \frac{23\pi}{12}$$

$$\tan \frac{23\pi}{12} = \frac{\sin \frac{23\pi}{12}}{\cos \frac{23\pi}{12}}$$

$$1) \sin \frac{23\pi}{12} = \sin \left( 2\pi - \frac{\pi}{12} \right)$$

$$\begin{aligned} \sin \left( 2\pi - \frac{\pi}{12} \right) &= \underbrace{\sin 2\pi}_{= 0} \cos \frac{\pi}{12} - \cos 2\pi \sin \frac{\pi}{12} \\ &= 0 - 1 \sin \frac{\pi}{12} \\ &= -\sin \frac{\pi}{12} \end{aligned}$$

$$\sin \frac{\pi}{12} = \sin \left( \frac{\pi}{4} - \frac{\pi}{6} \right)$$

$$\begin{aligned} \sin \left( \frac{\pi}{4} - \frac{\pi}{6} \right) &= \sin \frac{\pi}{4} \cos \frac{\pi}{6} - \cos \frac{\pi}{4} \sin \frac{\pi}{6} \\ &= \frac{1}{\sqrt{2}} \left( \frac{\sqrt{3}}{2} \right) - \left( \frac{1}{\sqrt{2}} \right) \left( \frac{1}{2} \right) \\ &= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \\ &= \frac{\sqrt{3} - 1}{2\sqrt{2}} \end{aligned}$$

$$\therefore \sin \frac{23\pi}{12} = - \left[ \frac{\sqrt{3} - 1}{2\sqrt{2}} \right]$$

$$\cos \frac{23\pi}{12} = \cos (2\pi - \frac{\pi}{12})$$

$$\begin{aligned}\cos (2\pi - \frac{\pi}{12}) &= \cos 2\pi \cos \frac{\pi}{12} + \sin 2\pi \sin \frac{\pi}{12} \\ &= (1) \cos \frac{\pi}{12} + (0) \\ &= \cos \frac{\pi}{12}.\end{aligned}$$

$$\cos \frac{\pi}{12} = \cos (\frac{\pi}{4} - \frac{\pi}{6})$$

$$\begin{aligned}\cos (\frac{\pi}{4} - \frac{\pi}{6}) &= \cos \frac{\pi}{4} \cos \frac{\pi}{6} + \sin \frac{\pi}{4} \sin \frac{\pi}{6} \\ &= \left(\frac{1}{\sqrt{2}}\right) \frac{\sqrt{3}}{2} + \left(\frac{1}{\sqrt{2}}\right) \left(\frac{1}{2}\right) \\ &= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \\ &= \frac{\sqrt{3} + 1}{2\sqrt{2}}\end{aligned}$$

$$\therefore \cos \frac{23\pi}{12} = \frac{\sqrt{3} + 1}{2\sqrt{2}}.$$

$$\begin{aligned}\therefore \tan \frac{23\pi}{12} &= \frac{-\left[\frac{\sqrt{3}-1}{2\sqrt{2}}\right]}{\frac{\sqrt{3}+1}{2\sqrt{2}}} \\ &= \frac{-\sqrt{3}+1}{2\sqrt{2}} \times \frac{2\sqrt{2}}{\sqrt{3}+1} = \frac{-\sqrt{3}+1}{\sqrt{3}+1}\end{aligned}$$