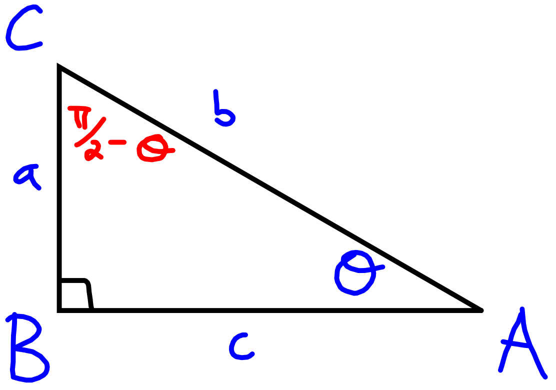
4.3: Equivalent Trigonometric Expressions

- Somemes, when trigonometry is used to model real-world applicaons, the expressions generated can become extremely complicated
- Equivalent trigonometric expressions are expressions that yield the same value for all values of the variable

Proving Cofuncon Idenes Involving $\frac{\pi}{2}-\theta$

We will use a right triangle to determine equivalent trigonometric expressions involving $\frac{\pi}{2}-\theta$



$$\measuredangle C = \pi - \left(\theta + \frac{\pi}{2}\right)$$
$$= \pi - \theta - \frac{\pi}{2}$$
$$= \frac{2\pi}{2} - \frac{\pi}{2} - \theta$$
$$\measuredangle C = \frac{\pi}{2} - \theta$$

Trig Ratios for θ:

$\sin\theta = \frac{a}{b}$; $\csc\theta = \frac{b}{a}$

$\cos\theta = \frac{c}{b}$; $\sec\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{c}$; $\cot\theta = \frac{c}{a}$

Trig Ratios for $\frac{\pi}{2}-\theta$

$\sin\left(\frac{\pi}{2}-\theta\right) = \frac{c}{b}$; $\csc\left(\frac{\pi}{2}-\theta\right) = \frac{b}{c}$

$\cos\left(\frac{\pi}{2}-\theta\right) = \frac{a}{b}$; $\sec\left(\frac{\pi}{2}-\theta\right) = \frac{b}{a}$

$\tan\left(\frac{\pi}{2}-\theta\right) = \frac{c}{a}$; $\cot\left(\frac{\pi}{2}-\theta\right) = \frac{a}{c}$

therefore:

$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$

$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$

$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$

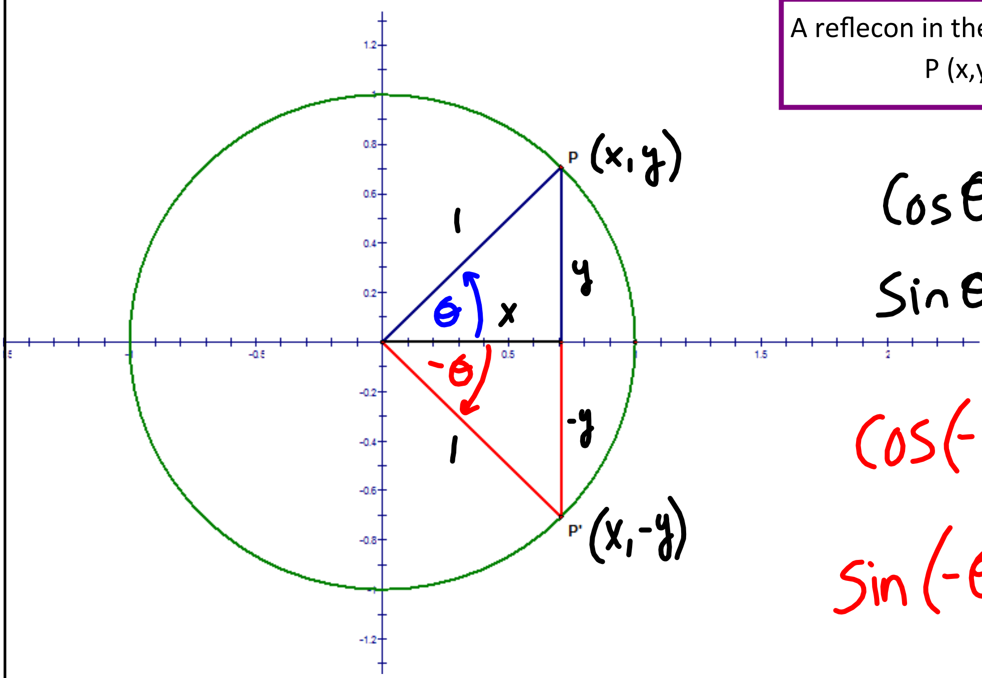
$\csc\left(\frac{\pi}{2}-\theta\right) = \sec\theta$

$\sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta$

$\cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$

Proving Cofuncon Idenes Involving $-\theta$

In the unit circle below, consider point P reflected in the x-axis



A reflecon in the x-axis will map point P (x,y) to P' (x, -y)

$\cos\theta = x$

$\sin\theta = y$

$\cos(-\theta) = x$

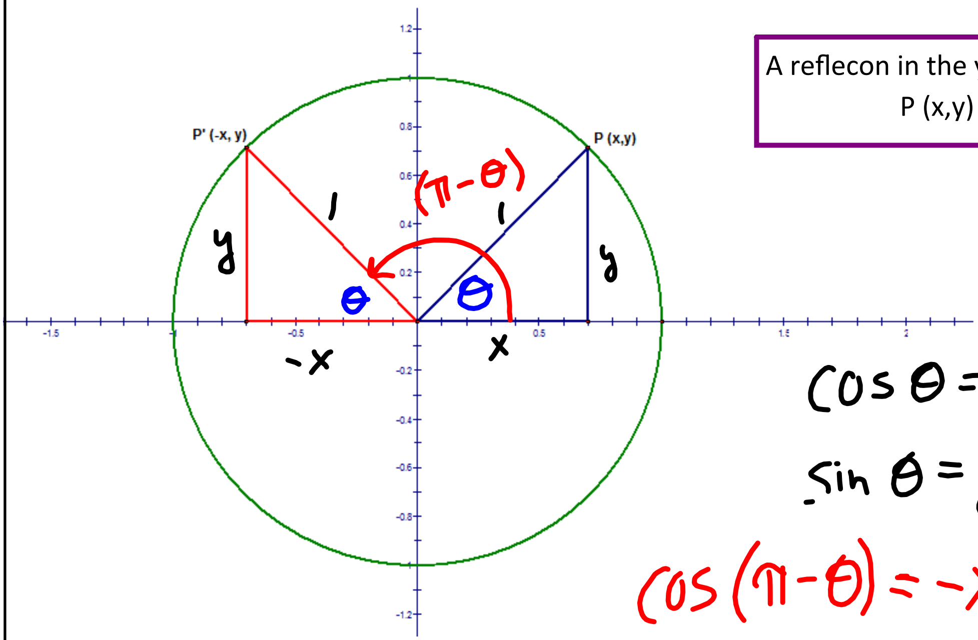
$\sin(-\theta) = -y$

$\cos(-\theta) = \cos\theta$

$\sin(-\theta) = -\sin\theta$

Proving Cofuncon Idenes Involving $\pi - \theta$

In the unit circle below, consider point P being reflected in the y-axis:



A reflecon in the y-axis will map point P (x,y) to P' (-x, y)

$\cos\theta = x$

$\sin\theta = y$

$\cos(\pi-\theta) = -x$

$\sin(\pi-\theta) = y$

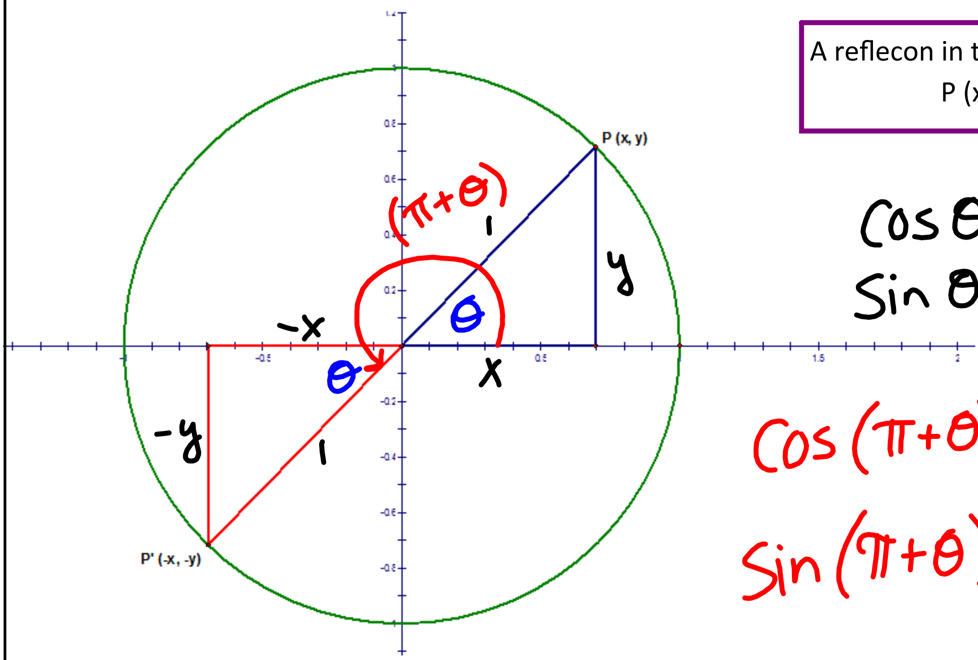
$\therefore$ $\cos(\pi-\theta) = -\cos\theta$

$\sin(\pi-\theta) = \sin\theta$

Proving Cofuncon Idenes Involving $\pi+\theta$

In the unit circle below, consider a reflecon of point P in the origin:

A reflecon in the origin will map point P (x,y) to P' (-x, -y)



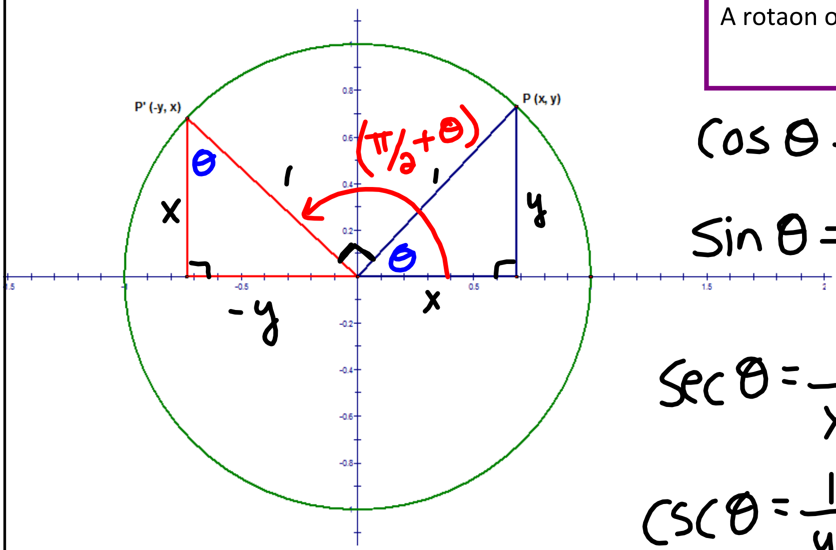
$$\cos\theta = x$$
$$\sin\theta = y$$

$$\cos(\pi+\theta) = -x$$
$$\sin(\pi+\theta) = -y$$

$$\cos(\pi+\theta) = -\cos\theta$$
$$\sin(\pi+\theta) = -\sin\theta$$

Proving Cofuncon Idenes Involving $\frac{\pi}{2}+\theta$

In the unit circle below, consider a rotaon of point P, by 90 degrees



A rotaon of 90 degrees will map point P (x,y) to P' (-y, x)

$\cos\theta = x$ $\quad \tan\theta = \frac{y}{x}$

$\sin\theta = y$

$\sec\theta = \frac{1}{x}$ $\quad \cot\theta = \frac{x}{y}$

$\csc\theta = \frac{1}{y}$

$\cos(\pi/2+\theta) = -y$ $\quad \sec(\pi/2+\theta) = \frac{-1}{y}$

$\sin(\pi/2+\theta) = x$ $\quad \csc(\pi/2+\theta) = \frac{1}{x}$

$\tan(\pi/2+\theta) = \frac{-x}{y}$ $\quad \cot(\pi/2+\theta) = \frac{-y}{x}$

$\cos(\pi/2+\theta) = -\sin\theta$ $\quad \sec(\pi/2+\theta) = -\csc\theta$

$\sin(\pi/2+\theta) = \cos\theta$ $\quad \csc(\pi/2+\theta) = -\sec\theta$

$\tan(\pi/2+\theta) = -\cot\theta$ $\quad \cot(\pi/2+\theta) = -\tan\theta$

Example 1) Use Equivalent Trigonometric Expressions to Evaluate Primary Trig Expressions

Given that $\sin\frac{\pi}{5} \doteq 0.5878$, use equivalent trigonometric expressions to evaluate the following, to four decimal places.

a) $\cos\frac{3\pi}{10}$

b) $\cos\frac{7\pi}{10}$

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a) $\frac{3\pi}{10} = 54°$, ∴ In the 1st Quadrant, so use a cofunction identity involving $(\pi/2 - \theta)$.

$\cos\left(\frac{3\pi}{10}\right) = \cos\left(\frac{\pi}{2} - \frac{\pi}{5}\right)$

apply co-function identity:

$\cos(\pi/2 - \theta) = \sin\theta$

$\cos\left(\pi/2 - \frac{\pi}{5}\right) = \sin(\pi/5)$

$\because \sin(\pi/5) \doteq 0.5878$

$\therefore \cos\left(\frac{3\pi}{10}\right) = \sin(\pi/5) \doteq 0.5878$

$\frac{3\pi}{10} = \frac{\pi}{2} - a$

$\frac{3\pi}{10} - \frac{\pi}{2} = -a$

$\frac{3\pi}{10} - \frac{5\pi}{10} = -a$

$\frac{-2\pi}{10} = -a$

$\frac{\pi}{5} = a$

(b) $\frac{7\pi}{10} = 126°$ ∴ In the 2nd quadrant, so use a cofunction identity involving $(\pi/2 + \theta)$

$\cos\left(\frac{7\pi}{10}\right) = \cos\left(\frac{\pi}{2} + \frac{\pi}{5}\right)$

apply cofunction identity:

$\cos(\pi/2 + \theta) = -\sin\theta$

$\frac{7\pi}{10} = \frac{\pi}{2} + \theta$

$\frac{7\pi}{10} - \frac{\pi}{2} = \theta$

$\frac{7\pi}{10} - \frac{5\pi}{10} = \theta$

$\frac{\pi}{5} = \theta$

$\cos\left(\frac{\pi}{2} + \frac{\pi}{5}\right) = -\sin(\pi/5)$

$\because \sin(\pi/5) \doteq 0.5878$

$\therefore \cos\left(\frac{7\pi}{10}\right) = -\sin(\pi/5) \doteq -0.5878$

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Example 2) Use an Equivalent Trigonometric Expressions to Evaluate a Reciprocal Trig Expression

Given that $\csc\frac{2\pi}{7} \doteq 1.2790$, use an equivalent trigonometric expression to determine $\sec\frac{3\pi}{14}$, to four decimal places

$\frac{3\pi}{14} \doteq 38.5°$, $\therefore$ angle is in Quadrant #1, so use a cofunction identity involving $(\pi/2 - \theta)$.

$$\frac{3\pi}{14} = \frac{\pi}{2} - \theta$$

$$\frac{3\pi}{14} - \frac{\pi}{2} = -\theta$$

$$\frac{3\pi}{14} - \frac{7\pi}{14} = -\theta$$

$$\frac{2\pi}{7} = \theta$$

$$\sec\left(\frac{3\pi}{14}\right) = \sec\left(\frac{\pi}{2} - \frac{2\pi}{7}\right)$$

apply the cofunction identity:

$$\sec(\pi/2 - \theta) = \csc\theta$$

$$\sec\left(\pi/2 - \frac{2\pi}{7}\right) = \csc(2\pi/7)$$

$$\therefore \csc\left(\frac{2\pi}{7}\right) \doteq 1.2790$$

$$\therefore \sec\left(\frac{3\pi}{14}\right) = \csc\left(\frac{2\pi}{7}\right) \doteq 1.2790$$

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