

4.1: Radian Measure

Determining the meaning of a radian measure: GSP Demo

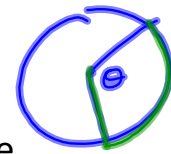
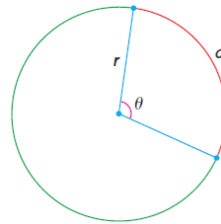
- Follow the investigation on page 203 - 204

Definition of a Radian

$$C = 2\pi r.$$

- The radian measure of an angle θ is defined as the length, a , of the arc that subtends the angle divided by the radius, r , of the circle.

$$\theta = \frac{a}{r}$$



- For one complete revolution, the length of the arc equals the circumference of the circle, $2\pi r$

$$\begin{aligned}\theta &= \frac{2\pi r}{r} \\ &= 2\pi\end{aligned}$$

$$\theta = \frac{a}{r} = \frac{2\pi r}{r}$$

$$\theta = 2\pi$$

- One complete revolution measures 2π radians.

$$\pi = 180^\circ$$

$$\begin{aligned}\frac{360^\circ}{2} &= \frac{2\pi}{2} \\ 180^\circ &= \pi\end{aligned}$$

Converng Between Radians and Degrees

$1 \text{ rad} = ?$

Converng Radians to Degrees

$$2\pi \text{ rad} = 360^\circ$$

$$1 \text{ rad} = \left(\frac{360}{2\pi} \right)^\circ$$

$$1 \text{ rad} = \left(\frac{180}{\pi} \right)^\circ$$

$1^\circ = \text{rad} ?$

Converng Degrees to Radians

$$360^\circ = 2\pi \text{ rad}$$

$$1^\circ = \frac{2\pi}{360} \text{ rad}$$

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

$2\pi = 360$

One radian is $\left(\frac{180}{\pi} \right)^\circ$, or approximately 57.3°. One degree is $\frac{\pi}{180}$ rad, or approximately 0.0175

Example 1: Convert 30° to radians

$$\begin{aligned} 30^\circ &\rightarrow \text{rad} \\ &= (30^\circ) \left(\frac{\pi}{180} \right) \\ &= \frac{30\pi}{180} \\ &= \frac{\pi}{6} \text{ rad or} \end{aligned}$$

$$\frac{\pi}{6} \rightarrow 0.52 \text{ rad}$$

Example 2: Converting Radian Measure to Degree Measure

Determine the degree measure, to the nearest tenth, for each radian measure.

(a) $\frac{\pi}{4}$

(b) 5.86

$$\begin{aligned} \text{(a)} \quad \frac{\cancel{\pi}}{4} \times \frac{180}{\cancel{\pi}} \\ &= \frac{180}{4} \\ &= 45^\circ \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 5.86 \times \frac{180}{\pi} \\ &= 5.86 \times 57.29577951 \\ &= 335.8^\circ \end{aligned}$$

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45° to radians

$$= 45^\circ \times \frac{\pi}{180}$$

$$= \frac{45\pi}{180}$$

$$= \frac{\pi}{4} \text{ rad.}$$

$$\frac{45\pi}{180}$$

$$\frac{45 \div 45}{180 \div 45} = \frac{1\pi}{4}$$

15° to radians:

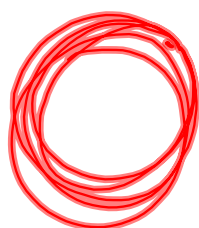
$$15^\circ \times \frac{\pi}{180}$$

$$= \frac{15\pi}{180}$$

$$= \frac{\pi}{12} \text{ rad}$$

Example 3: Arc Length for a Given Angle

Bruce Wayne chooses a horse to ride on a carousel. The horse is located 9 m from the centre of the carousel. If this carousel turns through an angle of 15π , determine the length of the arc travelled by the horse, to the nearest tenth of a metre.



$$\theta = \frac{a}{r} =$$

$$15\pi = \frac{a}{9}$$

$$(15\pi)9 = a$$

$$135\pi = a$$

$$\underline{424.1\text{m} = a}$$

Example 4: Angular Velocity of a Rotating Object

The angular velocity of a rotating object is the rate at which the central angle changes with respect to time.

The London Eye Ferris wheel has a diameter of 135 m and completes one revolution in 30 min.

(a) Determine the **angular velocity, ω** , in radians per second.

$$30 \text{ min} = \underline{1800 \text{ seconds.}} \quad 2\pi$$

$$\omega = \frac{1 \text{ revolution.}}{1800 \text{ seconds.}}$$

$$= \frac{2\pi}{1800} \quad \omega = \frac{\pi}{900}$$

(b) How far has a rider travelled at 10 min into the ride?

10 min = 600 seconds.

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$$\theta = (600 \text{ seconds}) \left(\frac{\pi}{900} \right)$$

$$= \frac{600\pi}{900}$$

$$\theta = \frac{2\pi}{3} \quad \theta = \frac{s}{r}$$

distance travelled:

$$s = \theta r$$

$$= \frac{2\pi}{3} (67.5)$$

$$= 141.4 \text{ m.}$$

Example 5: Angular Velocity of a Rotating Object

The hard disk in a personal computer rotates at 7200 rpm (revolutions per minute). Determine its angular velocity, in:

(a) Degrees per second

(b) Radians per Second.

