3.4: Solve Raonal Equaons and Inequalies

Solving Raonal Equaons:

- Raonal equaons can be solved algebraically
- To solve a raonal equaon algebraically, start by factoring the expressions in the numerator and denominator to find asymptotes and restricons.
- Next, mulply both sides by the **lowest common denominator**, and simplify to obtain a polynomial equaon.
- Then, solve the resulng polynomial equaon using the techniques learned in Chapter 2

Example 1)

Solve: $\frac{x+3}{x-4} = \frac{x-1}{x+2}$

$x \neq 4, -2$

$(x+2)(x+3) = (x-1)(x-4)$

$x^2+3x+2x+6 = x^2-4x-1x+4$

$x^2+5x+6 = x^2-5x+4$

$x^2-x^2+5x+5x+6-4=0$

$10x+2=0$

$\frac{\cancel{10}x}{\cancel{10}} = \frac{-2}{10}$

$x = \frac{-1}{5}$

$\frac{x+3}{x-4} = \frac{x-1}{x+2}$

$\frac{x+3}{x-4} - \left[\frac{x-1}{x+2}\right] = 0$

$\frac{(x+3)(x+2)}{(x-4)(x+2)} - \frac{(x-1)(x-4)}{(x-4)(x+2)} = 0$

$\frac{(x+3)(x+2) - [(x-1)(x-4)]}{(x-4)(x+2)} = \frac{0}{1}$

$0 = (x+3)(x+2) - [(x-1)(x-4)]$

$0 = x^2+5x+6 - [x^2-4x-1x+4]$

$0 = \cancel{x^2}+5x+6-\cancel{x^2}+4x+1x-4$

$0 = 10x+2$

$x = -1/5$

Example 2)

Solve: $\frac{x-5}{x^2-3x-4}=\frac{3x+2}{x^2-1}$

$x \neq -1, 4, +1$

$$\frac{x-5}{(x-4)(x+1)}=\frac{3x+2}{(x-1)(x+1)}$$

$$\frac{(x-4)\cancel{(x+1)}(3x+2)}{\cancel{(x+1)}}=\frac{(x-5)(x-1)\cancel{(x+1)}}{\cancel{(x+1)}}$$

$$(x-4)(3x+2)=(x-5)(x-1)$$

$$3x^2+2x-12x-8=x^2-x-5x+5$$

$$3x^2-10x-8=x^2-6x+5$$

$$3x^2-x^2-10x+6x-8-5=0$$

$$2x^2-4x-13=0$$

non-factorable, use quadratic formula:

$$=\frac{4 \pm \sqrt{(-4)^2-4(2)(-13)}}{2(2)}$$

$$=\frac{4 \pm \sqrt{120}}{4}$$

$$=\frac{4+10.95}{4}, \frac{4-10.95}{4}$$

$$=3.74, -1.74$$

Solving Raonal Inequalies

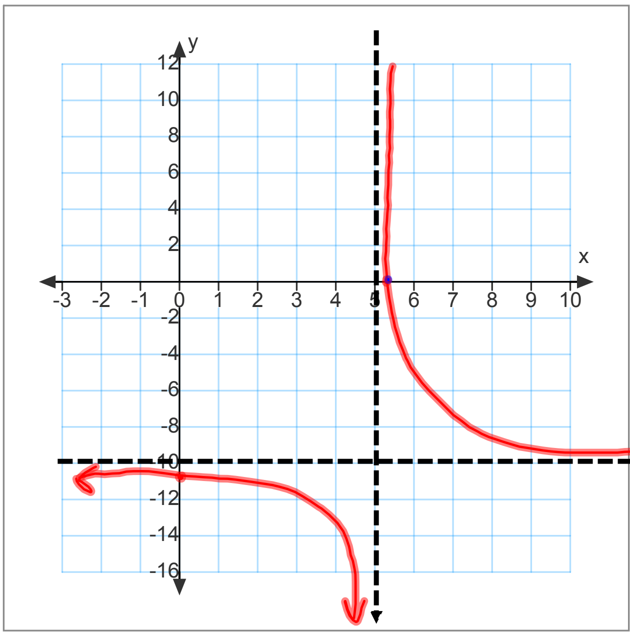
- Solving a raonal inequality means finding all the possible values of the variable that sasfy the inequality

To Solve a Raonal Inequality Algebraically

1. Rewrite the inequality with the right side equal to 0. This can be done by creang an equivalent polynomial inequality by mulplying all the terms by the LCD.
2. Using a chart (or number line), examine the posive and negave intervals for the equivalent polynomial inequality to determine the soluon.

Example 3a)

Solve graphically: $\frac{2}{x-5} < 10$



$$\frac{2}{x-5} - 10 < 0 \qquad \frac{1}{x}$$

$$\frac{2}{x-5} - \frac{10(x-5)}{(x-5)} < 0$$

$$\frac{2-10(x-5)}{x-5} < 0$$

$$\frac{2-10x+50}{x-5} < 0$$

$$\boxed{\frac{-10x+52}{x-5} < 0}$$

$$\frac{-2(5x-26)}{x-5} < 0$$

Sol'n: $x < 5$ and $x > 5.2$

Example 3b)

Solve algebraically: $\frac{2}{x-5} < 10$

$$\frac{-10x+52}{x-5} < 0$$

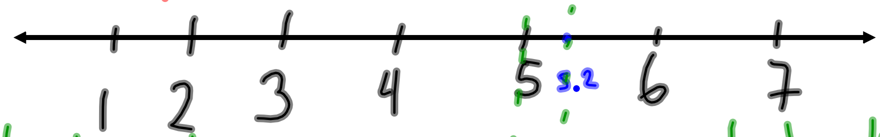
$$\frac{-2(5x-26)}{x-5} < 0$$

$5x-26=0$ $26/5$

$x = 26/5$

$(5, 5.2)$

$(-\infty, 5)$ $(5.2, \infty)$



test value $x=0$
y is "−"

test value
$x=5.1$
$y=$ "+"

test value
$x=10$
$y=$ "−"

$\therefore$ sol'n: $x<5$
$x>5.2$

Example 4)

Solve algebraically: $\dfrac{x+3}{x+1} \geq \dfrac{x-2}{x-3}$

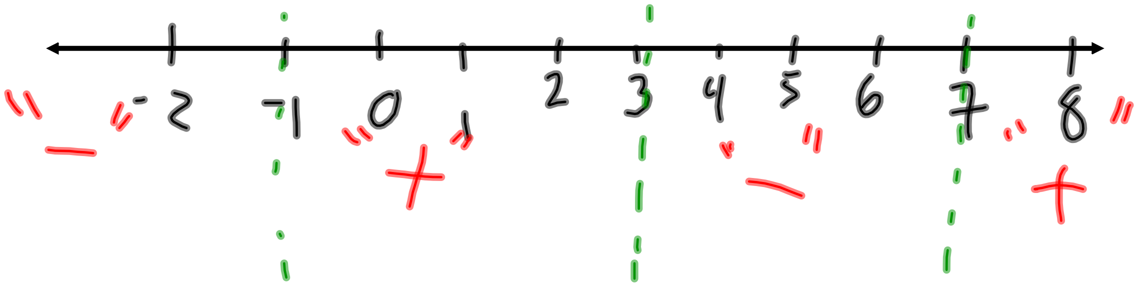
$$\frac{x-7}{(x+1)(x-3)} \geq 0$$

$$\frac{x+3}{x+1} - \left[\frac{(x-2)}{x-3}\right] \geq 0$$

$$\frac{(x+3)(x-3)}{(x+1)(x-3)} - \left[\frac{(x-2)(x+1)}{(x+1)(x-3)}\right] \geq 0$$

$$\frac{(x+3)(x-3) - [(x-2)(x+1)]}{(x+1)(x-3)}$$

$(-\infty, -1]$ $[-1, 3]$ $[3, 7]$ $[7, \infty)$



$\therefore$ sol'n $-1 < x \leq 3$ and $x \geq 7$

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