3.3: Raonal Funcons of the Form $f(x)=\dfrac{ax+b}{cx+d}$

Recall:

- A Raonal funcon is a funcon that can be expressed as $f(x)=\dfrac{p(x)}{q(x)}$ where *p(x)* and *q(x)* are polynomial funcons and $q(x)\neq 0$
- The funcon, $f(x)=\dfrac{3x^2-1}{x+1}$ $x\neq -1$ and $f(x)=\dfrac{1-x}{x^2}$ $x\neq 0$ are raonal funcons
- The funcon, $f(x)=\dfrac{1+x}{\sqrt{2-x}}$ $x\neq 2$ is not a raonal funcon because its denominator is not a polynomial

Sketching Raonal Funcons

- To sketch the graph of a raonal funcon we can use the domain, intercepts, equaons of asymptotes, and posive/negave intervals
- Depending upon the denominator of the raonal funcon, different types of disconnuites are possible

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Example 1)

Determine the key features of $f(x)=\frac{x-2}{3x+4}$. Use the key features of graph the funcon.

Soluon:

$f(x) = \frac{x-2}{3x+4}$

$3x+4=0$
$3x=-4$
$x=-4/3$

Domain: $\{x \mid x \neq -4/3,\ x \in \mathbb{R}\}$

Asymptotes:

Vercal Asymptote: $x = -4/3$

Horizontal Asymptote: To determine the horizontal asymptote, determine the value of *f(x)* as $x \to \pm\infty$

$f(x) = \frac{x-2}{3x-4}$

* divide every term by the largest degree of x.

$f(x) = \frac{\frac{x}{x} - \frac{2}{x}}{\frac{3x}{x} - \frac{4}{x}}$

$f(x) = \frac{1 - \frac{2}{x}}{3 - \frac{4}{x}}$ 0 0

$f(x) = \frac{1}{3}$

$f(x) = \frac{ax-b}{cx-d}$

The horizontal asymptote is the line $y = \frac{1}{3}$

* *Note: by examining the rao of the leading co-efficients **a** and **c** in the numerator and the denominator we can determine the equaon of the horizontal asymptote directly.*

$f(x) = \frac{x-2}{3x+4}$

Intercepts:

x-int, y=0

$0 = \frac{x-2}{3x+4}$

$0 = x-2$

$2 = x$

x -intercept: $x = 2$

y-int, x=0

$f(x) = \frac{0-2}{3(0)+4}$

$= \frac{-2}{4}$

$= -1/2$

y-intercept: $-1/2$ or -0.5

Posive/Negave Intervals:

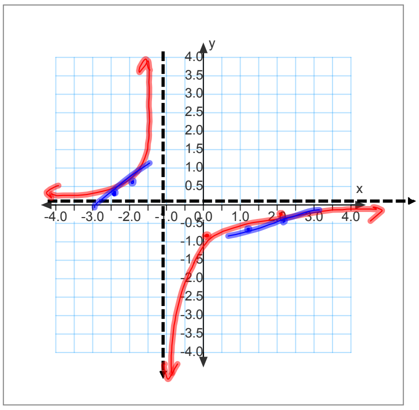
- The vercal asymptote and the x - intercept divides the set of real numbers into 3 intervals
- Choose numbers in each interval to evaluate the sign of the expression.
- You may decide to use an interval table to organize your work

Interval	x < -4/3	-4/3 < x < 2	x = 2	x > 2
f(x)	+	−	0	

Posive Interval(s): $(-\infty, -4/3)$; $(2, \infty)$

Negave Interval(s): $(-4/3, 2)$

Sketch:



Confirm the behaviour of f(x) near the vercal asymptote:

As $x \to \left(-\frac{4}{3}\right)^{+}, y \to$ $-\infty$

As $x \to \left(-\frac{4}{3}\right)^{-}, y \to$ $+\infty$

Intervals of Increase and Decrease

- The intervals of increase or decrease can be determined by analzying the graph of the funcon.
- It can also be determined by selecng 2 points in each interval and calculang the slope of the secant line. If the slope is **posive** , the funcon is **increasing**. If the slope is **negave** , the funcon is **decreasing**.

increasing from $(-\infty, \infty)$

Hole Disconnuies

- A raonal funcon, $f(x)=\frac{p(x)}{q(x)}$ has a hole at $x = a$, if $\frac{p(a)}{q(a)}=\frac{0}{0}$. This occurs when $p(x)$ and $q(x)$ contain a common factor of $(x - a)$.

Example 2)

$g(3)=\frac{0}{0}$ $g(x)=\frac{\cancel{(x-3)}}{2\cancel{(x-3)}}$ $g(x)=\frac{1}{2}$

Given: $g(x)=\frac{x-3}{2x-6}$

(i) Determine the domain, intercepts, asymptotes, and posive/negave intervals

(ii) Use these characteriscs to sketch the graph for $g(x)$

(iii) Describe where the funcon is increasing or decreasing

Soluon:

(i) domain: $\{x \mid x \neq 3, x \in \mathbb{R}\}$

intercepts: x-int, y=0

$\frac{0}{1}=\frac{x-3}{2x-6}$ $0=x-3$ $3=x$ ∴ no x-int, b/c a hole exists @ x=3

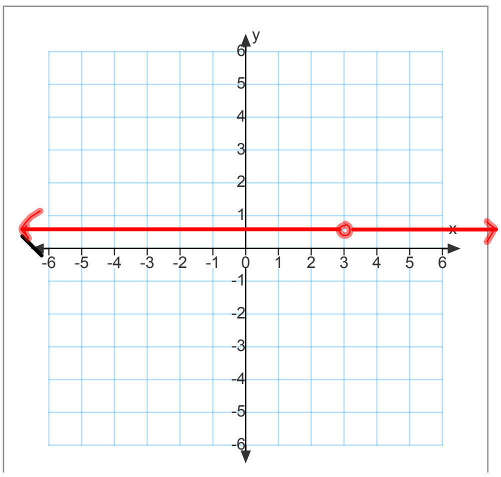
y-int, x=0

$y=\frac{0-3}{2(0)-6}$
$=\frac{-3}{-6}$
$=1/2$

y-int is 1/2

Asymptotes: - no vertical asymptote b/c a hole exists at x=3
- no horizontal asymptote.

positive throughout the entire domain.



Example 2)

Given:

(i) Determine the domain, intercepts, asymptotes, and posive/negave intervals

(ii) Use these characteriscs to sketch the graph for *g(x)*

(iii) Describe where the funcon is increasing or decreasing

Soluon: