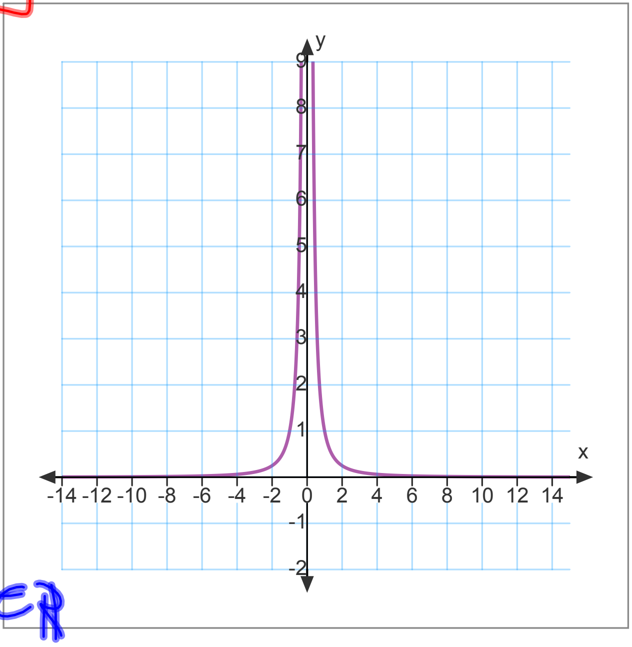
3.2: Reciprocal of a Quadrac Funcon

Raonal Funcon:

- Raonal funcons can have polynomials of any degree, such as quadrac, cubic, quarc, quinc, etc; in the numerator and denominator
- Quadracs have possibly zero, one, or two x-intercepts, a parabolic shape, and a maximum or minimum point. As a result of this, plong their reciprocals can be complex.
- Raonal funcons can be analzyed using key features: asymptotes, intercepts, slope (posive or negave, increasing or decreasing), domain, range, and posive and negave intervals.

Invesgate the graph of $\boxed{y=\frac{1}{x^2}}$

$y=\frac{1}{x^2}$



- Horizontal Asymptote:

  $y=0$

- Vercal Asymptote:

  $x=0$

- Intercepts:

  no intercepts

- Domain and Range:

  $D: x \in \mathbb{R}, x \neq 0$ $R: y>0, y \in \mathbb{R}$

- End Behaviour

  $x \to \infty, y \to 0; x \to -\infty, y \to 0$

- Posive and Negave Intervals:

  positive always

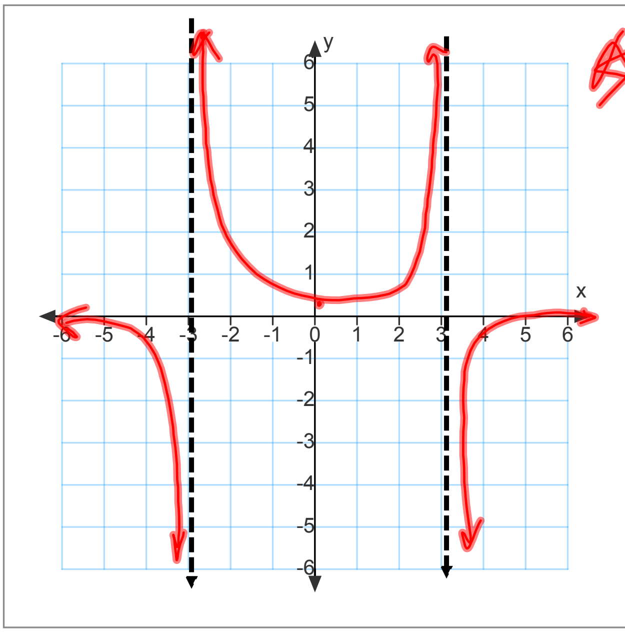
- Increasing and Decreasing intervals

  increasing $(-\infty, 0)$
  decreasing $(0, \infty)$

The Nature of the Graph of the Reciprocal of a Quadrac Funcon

Consider the funcon $f(x)=\dfrac{1}{9-x^2}$

$f(x)=\dfrac{1}{(3-x)(3+x)}$



(a) Determine the vercal asymptotes:

$x=3$ and

$x=-3$

- There will be a maximum or minimum between the asymptotes. Just as a quadrac, the reciprocal of a quadrac has the **x-coordinate** of its maximum or minimum **exactly halfway** between the vercal asymptotes

$x=\dfrac{3+(-3)}{2}$
$=0$

$f(0)=\frac{1}{9}=0.\bar{1}$

- There is a local minimum at: ___________________ $y=0.1$

(b) Determine the domain and range:

Domain: $\{x\in\mathbb{R}, x\neq 3,-3\}$

Range: $\{y\in\mathbb{R}, y\geq 1/9, y\neq 0\}$

$y=\dfrac{1}{(9-x^2)}$

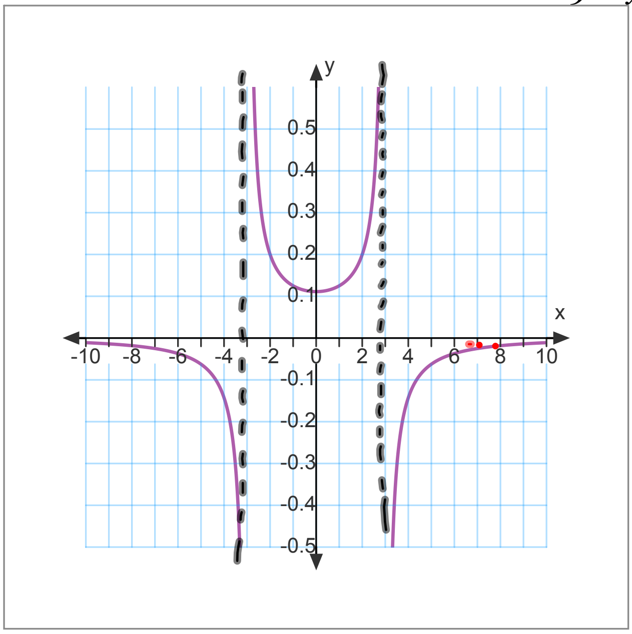
$=\dfrac{1}{9}$

(c) Determine the x- and y- intercepts:

x-intercept: no x-int!

y-intercept: $y=1/9$

(d) Determine the behaviour of the funcon near the **asymptotes**: $y = \frac{1}{9 - x^2}$



- As $x \to 3^+, y \to$ $-\infty$
- As $x \to 3^-, y \to$ $+\infty$
- As $x \to -3^+, y \to$ $+\infty$
- As $x \to -3^+, y \to$ $-\infty$

(e) Determine the **end behaviour**:

- As $x \to \infty, y \to$ 0
- As $x \to -\infty, y \to$ 0
- There is a **horizontal asymptote**, at $y=0$

(f) Determine **posive and negave** intervals:

f(x) is posive when $-3 < x < 3$ or $(-3, 3)$

f(x) is negave when $-\infty < x < -3$ and $3 < x < \infty$

(g) Determine **increasing** and **decreasing** intervals

- Increasing intervals means as $x\uparrow, y\uparrow$
- Decreasing intervals means as $x\uparrow, y\downarrow$

- *f(x)* is increasing when $0 < x < 3$ and $3 < x < \infty$
- *f(x)* is decreasing when $-\infty < x < -3$ and $-3 < x < 0$

Example: Sketch $f(x) = \dfrac{-1}{2x^2 - 1x - 6}$

$\dfrac{-1}{18+3-6}$

$f(x) = \dfrac{-1}{(2x+3)(x-2)}$

Soluon: Analyze *f(x)* using the key features:

- Domain
- Range
- Asymptotes
- Intercepts
- End Behaviour
- Behaviour of *f(x)* near the asymptotes
- Posive and Negave Intervals
- Increasing and Decreasing Intervals
- Max and Min Points

Asymptotes:

$x = -3/2$

$x = 2$

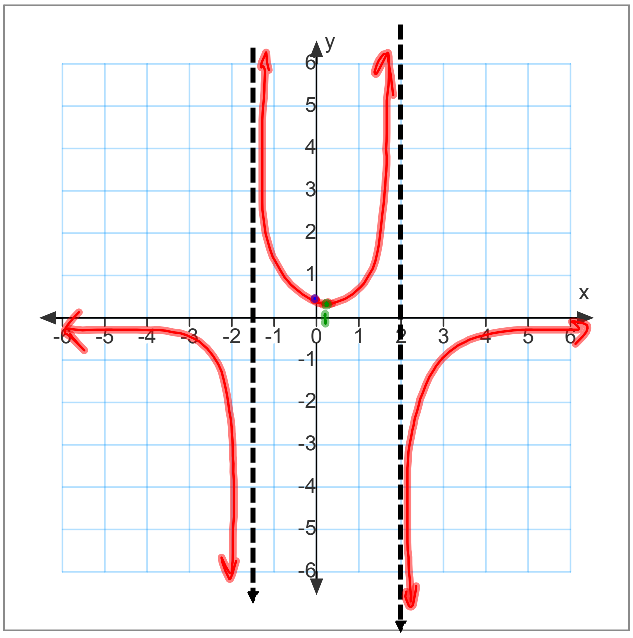
Intercepts

y-int: $y = \frac{-1}{(3)(-2)} = \frac{1}{6}$

Local /max/min:

$\frac{-1.5+2}{2} = 0.25$

$f(0.25) = 0.1633$



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