

### 3.1: Reciprocal of a Linear Funcon

#### Raonal Funcon:

- A funcon of the form  $\frac{p(x)}{q(x)}$  where  $p(x)$  and  $q(x)$  are both polynomial funcons,  $q(x)$  cannot be zero.

#### Asymptote:

- A line that a curve approaches more and more closely but does **not** intersect. The line may be a vercal asymptote, a horizontal asymptote, or an oblique asymptote.

#### Reciprocal Funcon:

- If  $y = g(x)$ , then its reciprocal funcon is  $f(x) = \frac{1}{g(x)}$ . When  $g(x) = 0$ , then there is a vercal asymptote for  $f(x)$ .

Example 1: Characteristics of a Reciprocal of a Linear Function

Consider the function  $f(x) = \frac{1}{2x+1}$

$f(x) = \frac{1}{g(x)}$   
 $g(x) = 2x+1$   
 $0 \neq 2x+1$

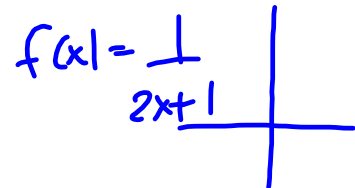
(a) State the domain

$D: \{x \in \mathbb{R}, x \neq -\frac{1}{2}\}$

$-\frac{1}{2} = x$   $\frac{1}{x}$

(b) Determine the x-intercept

$\frac{0}{1} = \frac{1}{2x-1}$  no x-int.  
 $0 = 1$  } not possible.

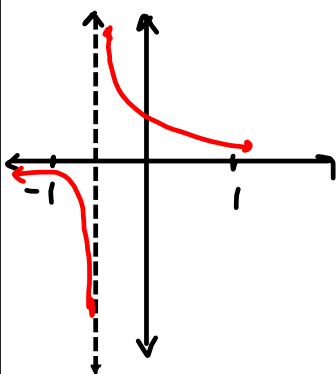


(c) Determine the y-intercept

$f(x) = \frac{1}{2x+1}$   $f(0) = \frac{1}{2(0)+1} = +1$  y-int is +1.

(d) Describe the behaviour of the function near the vertical asymptote

$f(x) = \frac{1}{2x+1}$  vertical asymptote at  $x = -\frac{1}{2}$



$x \rightarrow -\frac{1}{2}^+$ ,  $y \rightarrow +\infty$

$x \rightarrow -\frac{1}{2}^-$ ,  $y \rightarrow -\infty$

w/o graph,

$x \rightarrow -\frac{1}{2}^+$   $f(0) =$   
 $f(-0.25) =$   
 $f(-0.49) =$

$x \rightarrow -\frac{1}{2}^-$   $f(-1) =$   
 $f(-3/4) =$   
 $f(-0.51) =$

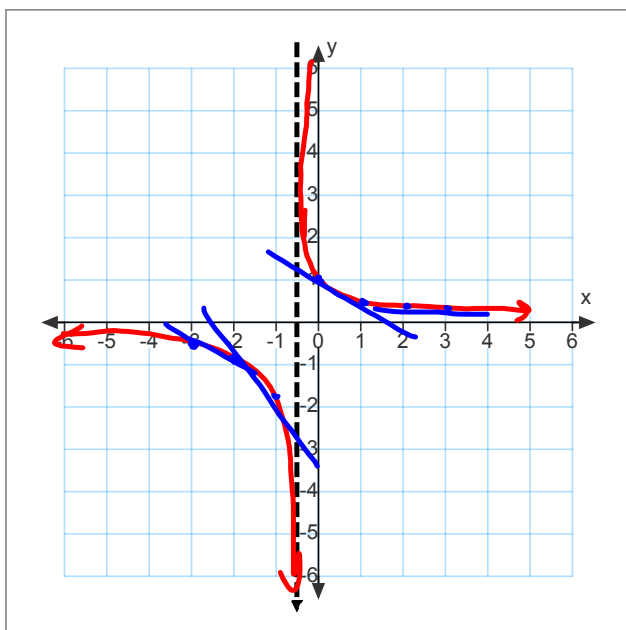
(e) Describe the end behaviour

$$x \rightarrow \infty, y \rightarrow 0$$

$$x \rightarrow -\infty, y \rightarrow 0$$

$\therefore$  There is a horizontal asymptote at  $y=0$

(f) Sketch the graph of the function



$$y = \frac{1}{x}$$

$$D: \{x \in \mathbb{R}, x \neq 0\}$$

$$R: \{y \in \mathbb{R}, y \neq 0\}$$

(g) State the range

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(h) Describe the intervals where the slope is increasing and the intervals where the slope is decreasing in the two branches of  $f(x)$ .

Slope is increasing on the interval  $(-\frac{1}{2}, \infty)$ .

Slope is decreasing on the interval  $(-\infty, -\frac{1}{2})$ .

Pg 153-154

# 1-9