2.6: Solve Factorable Polynomial Inequalies Algebraically

Recall: Solve Linear Inequalies

Solve each linear inequality. Show the soluon on a number line

(a) $x - 8 \geq 3$

(b) $-4 - 2x < 12$

(a) $x - 8 \geq 3$

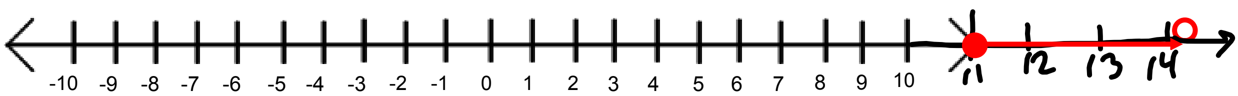
$x \geq 3 + 8$

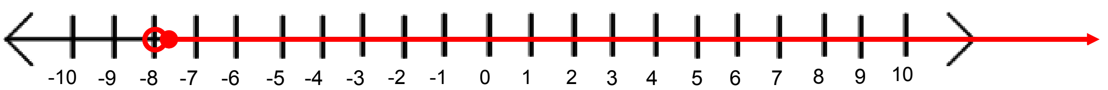
$x \geq 11$

$-2x < 12 + 4$

$\frac{-2x}{-2} < \frac{16}{-2}$

$x > -8$





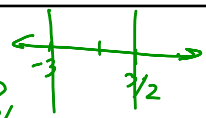
Example 2: Solve Polynomial Inequalies Algebraically

Solve each inequality

(a) (x + 3)(2x - 3) > 0

(b) $-2x^3 - 6x^2 + 12x + 16 \le 0$

$x = -3$
$x = 3/2$



Soluon for (a)

Method 1: ***Consider All Cases***

$\underbrace{(x+3)}_{m}^{+}\underbrace{(2x-3)}_{n}^{-} > 0$

A product *mn* is posive when *m* and *n* are:

- Both posive
- Both negave

Case #1 : both positive

$x + 3 > 0$
$x > 0 - 3$
$x > -3$

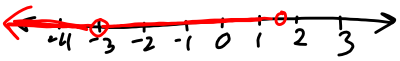
$2x - 3 > 0$
$2x > 3$
$x > 3/2$

Case #2 : Both " — "

$x + 3 < 0$
$x < -3$

$2x - 3 < 0$
$2x < 3$
$x < 3/2$

∴ Sol'ns $x < -3$, $x > 3/2$



Soluon for (a) connued

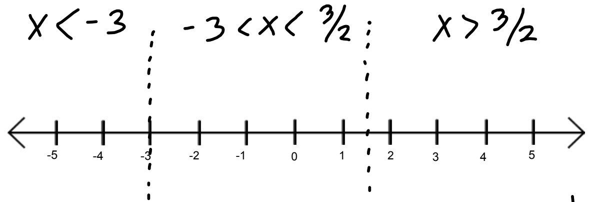
Method 2: ***Use Intervals***

(x + 3)(2x - 3) > 0

The roots of the equaon (x +3)(2x - 3) = 0 are x = -3 and x = 3/2.

Use the roots to break the number line into three intervals:

$x < -3$; $-3 < x < 3/2$; $x > 3/2$



Test arbitrary values of *x* for each interval:

Test $x < -3$
Choose $x = -4$
$(2(-4) - 3)(-4 + 3)$
$(-11)(-1) = +11$

Test $-3 < x < 3/2$
Choose $x = 0$
$(2(0) - 3)(0 + 3)$
$(-3)(3)$
-9

Test $x > 3/2$
Choose $x = 2$
$(2(2) - 3)(2 + 3)$
$(1)(5)$
$= 5$

Summarize the results in a table:

$(x+3)(2x-3) > 0$

	x < -3	x = -3	-3 < x < 3/2	x = 3/2	x > 3/2
(x + 3)	−	0	+	+	+
(2x - 3)	−	−	−	0	+
(x + 3)(2x - 3)	+	0	−	0	+

Show soluon on a number line:

$-2x^3 - 6x^2 + 12x + 16 \le 0$ $-2(x^3 + 3x^2 - 6x - 8) \le 0$

Soluon for (b)

- First, factor the polynomial expression to determine the roots

$-2x^3 - 6x^2 + 12x + 16 = -2(x+4)(x-2)(x+1)$

$\underbrace{-2(x+4)}_{a}\underbrace{(x-2)}_{b}\underbrace{(x+1)}_{c} \le 0$

$a\ b\ c \le 0$

Method 1: ***Consider All Cases***

- Since -2 is a constant factor, it can be combined with (x + 4) to form one factor
- The three factors of -2(x + 4)(x - 2)(x + 1) are: -2(x + 4); x - 2; and x + 1
- A product *abc* is negave when:
  - > all three factors, *a, b,* and *c,* are negave
  - > two of the factors are posive and the third one is negave
- This makes cases to consider.

Case 1

$-2(x+4) \le 0$

$-2x - 8 \le 0$

$-2x \le 8$

$x \ge -4$

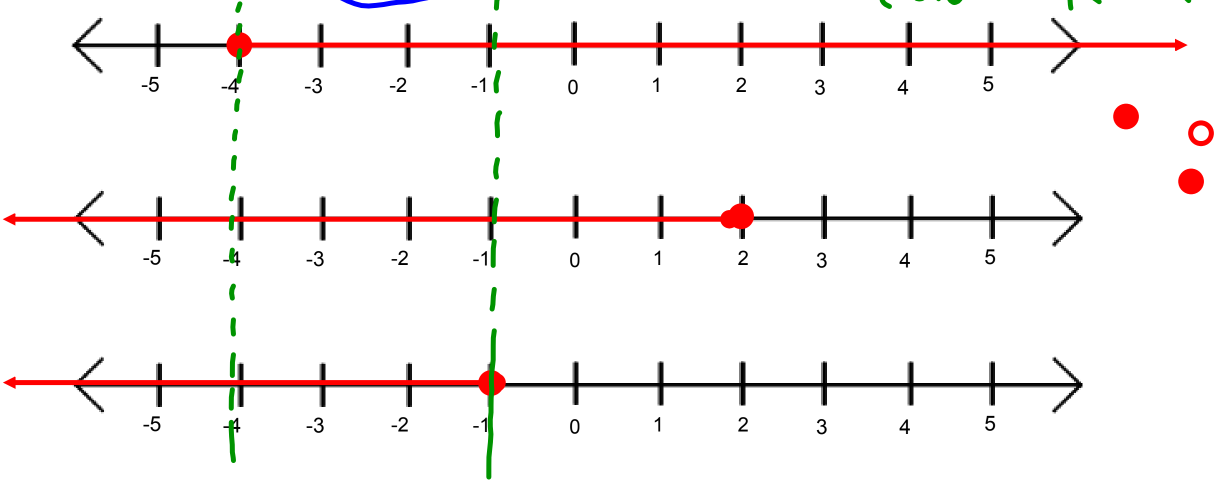
$x - 2 \le 0$

$x \le 2$

$x + 1 \le 0$

$x \le -1$

Soln for this case: $-4 \le x \le -1$



Case 2

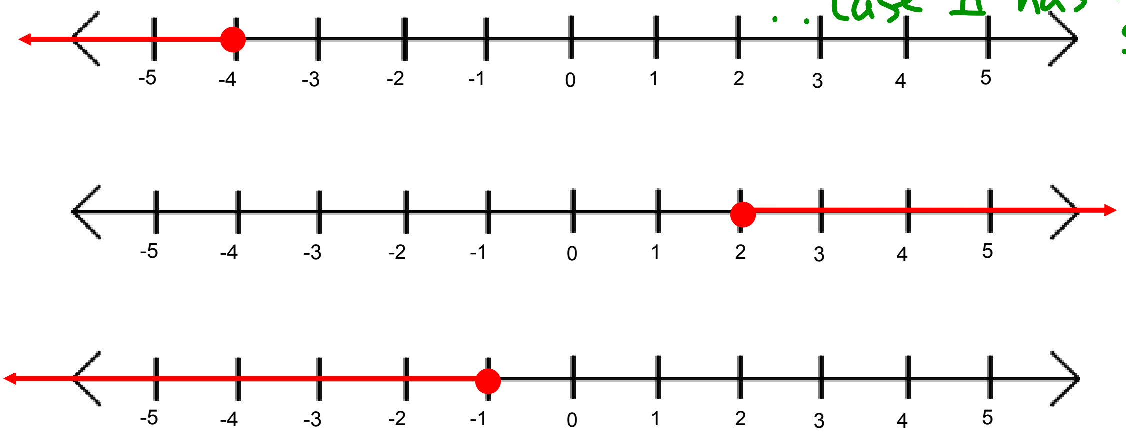
$$-2(x+4)(x-2)(x+1) \leq 0$$

$-2x-8 \geq 0$; $x-2 \geq 0$; $x+1 \leq 0$

$-2x \geq 8$; $x \geq 2$; $x \leq -1$

$x \leq -4$

no x-values in common, $\therefore$ Case II has no sol'n!



Case 3

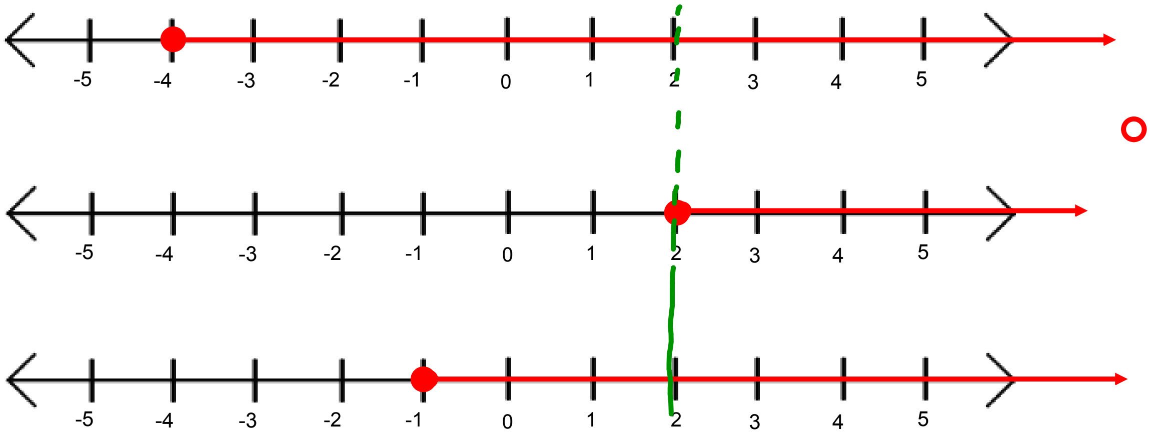
$$-2(x+4)(x-2)(x+1) \leq 0$$

$-2x-8 \leq 0$; $x-2 \geq 0$; $x+1 \geq 0$

$-2x \leq 8$ $\quad$ $x \geq 2$ $\quad$ $x \geq -1$

$x \geq -4$

$\therefore x \geq 2$



Case 4 $-2(x+4)(x-2)(x+1) \leq 0$

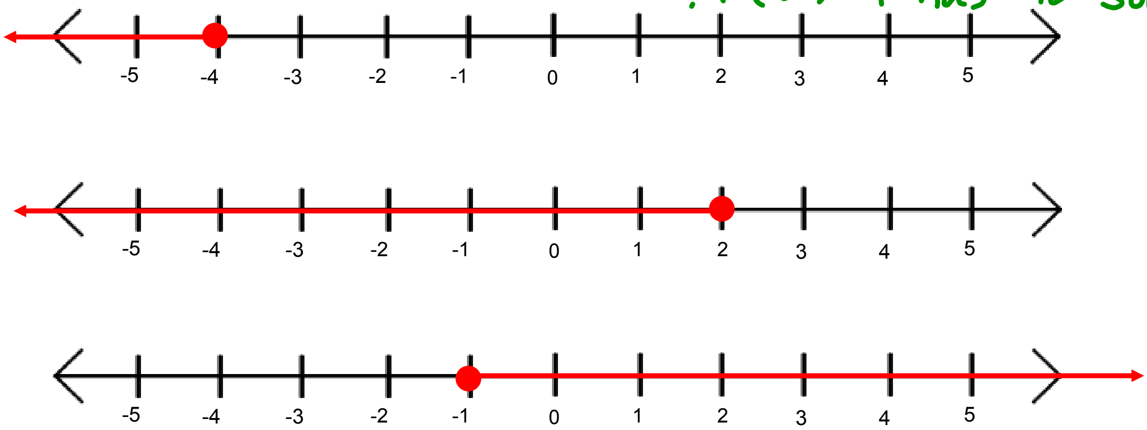
$-2x-8 \geq 0$; $x-2 \leq 0$; $x+1 \geq 0$

$-2x \geq 8$ $x \leq 2$ $x \geq -1$

$x \leq -4$

∴ no x-values in common ∴

∴ Case 4 has no sol'n ∴



∴ looking at all the cases, the sol'n to the inequality: $-2(x+4)(x-2)(x+1) \leq 0$

is: $-4 \leq x \leq -1$ and $x \geq 2$

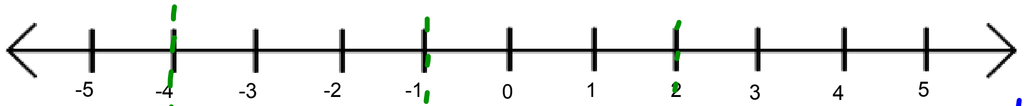
Soluon for (b)

Method 2: ***Use Intervals***

$$-2x^3 - 6x^2 + 12x + 16 = -2(x+4)(x-2)(x+1)$$
$$-2(x+4)(x-2)(x+1) \le 0$$

- The roots of the equaon -2(x + 4)(x - 2)(x + 1) = 0 are *x* = -4, *x* = -1 and *x* = *2*
- Use the roots to break the number line into 4 intervals
- Test arbitrary values of *x* in each interval.

$-2(x+4)(x-2)(x+1) \le 0$



test $x=-5$
$-2(-5+4)(-5-2)(-5+1)$
$-2(-1)(-7)(-4)$
$+$

test $x=-2$
$f(x)=$ "−"

test $x=0$
$f(x)=$ "+"

test $x=3$
$f(x)=$ "−"

	x < -4	x = -4	-4 < x < -1	x = -1	-1 < x < 2	x = 2	x > 2
-2(x + 4)	+	0	−	−	−	−	−
(x - 2)	−	−	−	−	−	0	+
(x + 1)(2x - 3)	−	−	−	0	+	+	+
-2(x + 4)(x - 2)(x + 1)	+	0	−	0	+	0	−

Sol'n: $-4 \le x \le -1$ and $x \ge 2$