2.4: Families of Polynomial Funcons

- The graphs of polynomial funcons that belong to the same family have the same x-intercepts but have different y-intercepts (unless zero is one of the intercepts).
- An equaon for the family of polynomial funcon with zeros a_1, a_2, a_3, ..., a_n is $y = k(x - a_1)(x - a_2)(x - a_3)\ldots(x - a_n)$ where $k \in \mathbb{R}, k \neq 0$

Example 1: Represent a Family of Funcons Algebraically

The zeros of a family of quadrac funcons are 2 and -3.

(a) Determine an equaon for this family of funcons

(b) Write equaons for two funcons that belong to this family.

(c) Determine an equaon for the member of the family that passes through the point (1, 4).

(a) $f(x) = k(x-2)(x+3)$

(b) $k = 4$

$f(x) = 4(x-2)(x+3)$

$k = -1$

$f(x) = -(x-2)(x+3)$

(c) $f(x) = k(x-2)(x+3)$

$4 = k(1-2)(1+3)$

$4 = k(-1)(4)$

$4 = -4k$

$\therefore k = -1$

$\therefore$ eqn:

$f(x) = -1(x-2)(x+3)$

Example 2: Determine an Equaon for a Family of Cubic Funcons Given Integral Zeros

The zeros of a family of cubic funcons are -2, 1, and 3

(a) Determine an equaon for this family

(b) Write equaons for two funcons that belong to this family

(c) Determine an equaon for the member of the family whose graph has a y-intercept of -15

(d) Sketch graphs of the funcons in parts b) and c)

(a) $f(x) = k(x+2)(x-1)(x-3)$

(b) $k = 2$

$f(x) = 2(x+2)(x-1)(x-3)$

if $k = -3$

$f(x) = -3(x+2)(x-1)(x-3)$

(c) $(0,-15)$ is on the graph

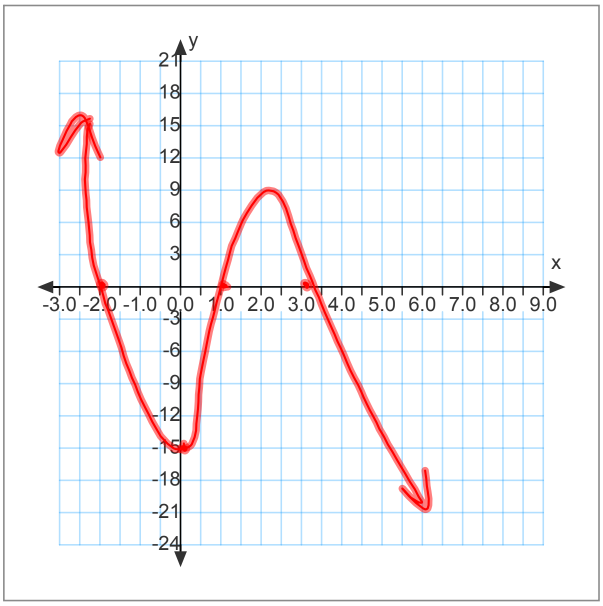
$f(x) = k(x+2)(x-1)(x-3)$

$-15 = k(0+2)(0-1)(0-3)$

$-15 = k(2)(-1)(-3)$

$\frac{-15}{6} = \frac{6k}{6}$ $k = \frac{-5}{2}$

$\therefore$ eqn: $f(x) = \frac{-5}{2}(x+2)(x-1)(x-3)$



Example 3 Determine an Equaon for a Family of Quarc Funcons Given Irraonal Zeros

(a) Determine a simplified equaon for the family of quarc funcons with zeros ± 1 and $2 \pm \sqrt{3}$

(b) Determine an equaon for the member of the family whose graph passes through the point (2, 18)

(a) zeros: $+1$, -1, $2+\sqrt{3}$, and $2-\sqrt{3}$

$f(x) = k(x-1)(x+1)(x-(2+\sqrt{3}))(x-(2-\sqrt{3}))$

$f(x) = k(x-1)(x+1)(x-2-\sqrt{3})(x-2+\sqrt{3})$

$(2, 18)$

$18 = k(2-1)(2+1)(2-2-\sqrt{3})(2-2+\sqrt{3})$

$18 = k(1)(3)(-\sqrt{3})(+\sqrt{3})$

$18 = -9k$

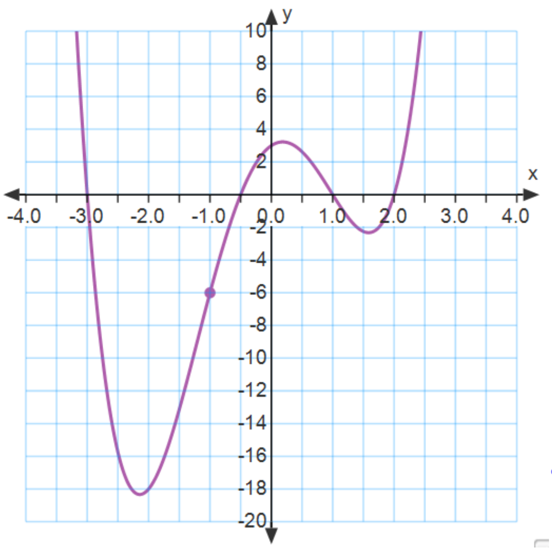
$\frac{18}{-9} = \frac{-9k}{-9}$

$k = -2$

$\therefore f(x) = -2(x-1)(x+1)(x-2-\sqrt{3})(x-2+\sqrt{3})$

Example 4: Determine an Equaon for a Quarc Funcon from a Graph

Determine an equaon for the quarc funcon represented by this graph



Zeros: $-3, -\frac{1}{2}, 1, 2$

$f(x) = k(x+3)(2x+1)(x-1)(x-2)$

$-6 = k(-1+3)(2(-1)+1)(-1-1)(-1-2)$

$-6 = k(2)(-1)(-2)(-3)$

$\frac{-6}{-12} = \frac{-12k}{-12}$ $k = \frac{1}{2}$

$\therefore$ eqn: $f(x) = \frac{1}{2}(x+3)(2x+1)(x-1)(x-2)$

Key Ideas:

- The real roots of a polynomial equaon *P(x) = 0* correspond to the x-intercepts of the graph of the polynomial funcon *P(x)*
- The x-intercepts of the graph of a polynomial funcon correspond to the real roots of the related polynomial equaon
- If a polynomial equaon is factorable, the roots are determined by factoring the polynomial, seng its factors equal to zero, and solving each factor
- If a polynomial equaon is not factorable, the roots can be determined from the graph using technology.