2.3: Polynomial Equaons

Recall:

- To solve a quadrac equaon such as $2x^2 - 12 = -5x$ we first bring everything over to one side, then factor the quadrac to determine its factors, then use the zero product theorem to determine the roots.
- The same principles apply in solving a polynomial equaon
  - > Bring everything over to 1 side
  - > Factor the polynomial equaon using factor theorem, polynomial division, etc.
  - > Determine the roots using the zero product theorem

Example 1: Solve Polynomial Equaons by Factoring and Factor Theorem

Solve:

(a) $(3x^3 + x^2) - (12x - 4) = 0$

(b) $2x^3 + 3x^2 - 11x - 6 = 0$

(a) $x^2(3x+1) - 4(3x+1) = 0$

$(3x+1)(x^2-4) = 0$

$(3x+1)(x-2)(x+2) = 0$

$\therefore$ roots $= -1/3, 2, -2$

(b) $2x^3 + 3x^2 - 11x - 6 = 0$

b: $\pm 1, \pm 2, \pm 3, \pm 6$

a: $\pm 1, \pm 2$

$\frac{b}{a}$: $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 6$

$f(2) = 0 \therefore x-2$ is a factor.

$$\begin{array}{r|rrrr} 2 & 2 & 3 & -11 & -6 \\ & & 4 & 14 & 6 \\ \hline & 2 & 7 & 3 & 0R \end{array}$$

$(x-2)(2x^2+7x+3)$

$(x-2)(2x^2+1x+6x+3)$

$(x-2)(x(2x+1)+3(2x+1))$

$(x-2)(2x+1)(x+3) = 0$

$\therefore x = 2, -3, -1/2$

Note:

- Some polynomial equaons may have real and non-real roots.
  Consider the soluon to the polynomial equaon: $(x-3)(x^2+1)=0$

$(-3)(+1)=$

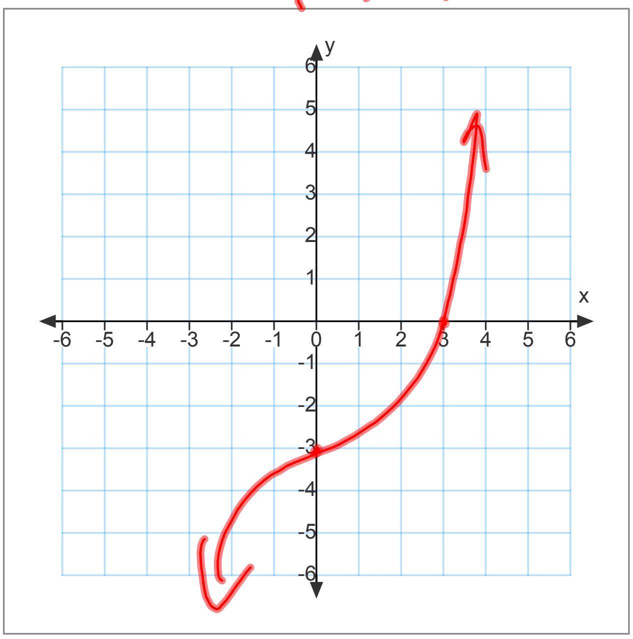
$X=3$

$(x^2+1)=0$

$x^2=-1$

$X=\pm\sqrt{-1}$

not possible!



Factoring a Sum or Difference of Cubes

Sum of Cubes

- An expression that contains two perfect cubes that are added together is called a ***sum of cubes*** and can be factored as follows:

  $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

- An expression that contains perfect cubes where one is subtracted from the other is called a ***difference of cubes*** and can be factored as follows:

  $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

Example 2: Factor a sum or difference of cubes

Factor the following expressions:

(a) $27x^3 + 125$

(b) $7x^4 - 448$

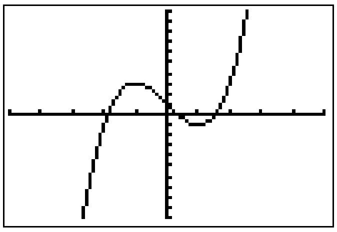
(c) $x^9 - 512$

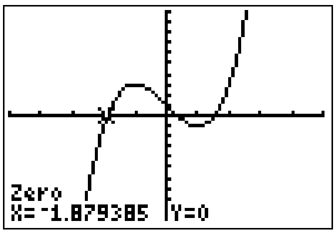
Example 3: Determining the Roots of a Non-Factorable Polynomial Eqn

- Some polynomial equaons are non-factorable. If this occurs, the roots can be found using graphing technology. In the case of a non-factorable polynomial equaon, the zeros of the funcon are non-integer values.

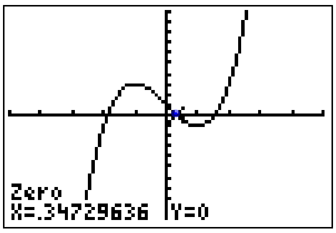
(a) Solve $x^3 - 3x = -1$. Round the roots to one decimal place.

$$x^3 - 3x + 1 = 0$$





-1.9



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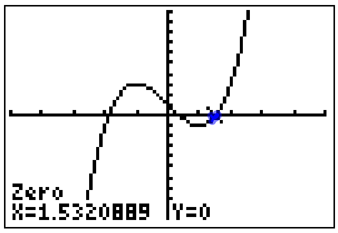

1.5

Key Ideas:

- The real roots of a polynomial equaon $P(x) = 0$ correspond to the x-intercepts of the graph of the polynomial funcon $P(x)$
- The x-intercepts of the graph of a polynomial funcon correspond to the real roots of the related polynomial equaon
- If a polynomial equaon is factorable, the roots are determined by factoring the polynomial, seng its factors equal to zero, and solving each factor
- If a polynomial equaon is not factorable, the roots can be determined from the graph using technology.

P.g # 110-112
#1-9 (pick n choose)
#10-15
#17 and #19