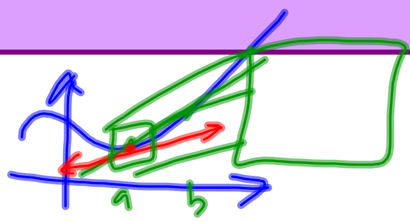
1.6: Instantaneous Rates of Change

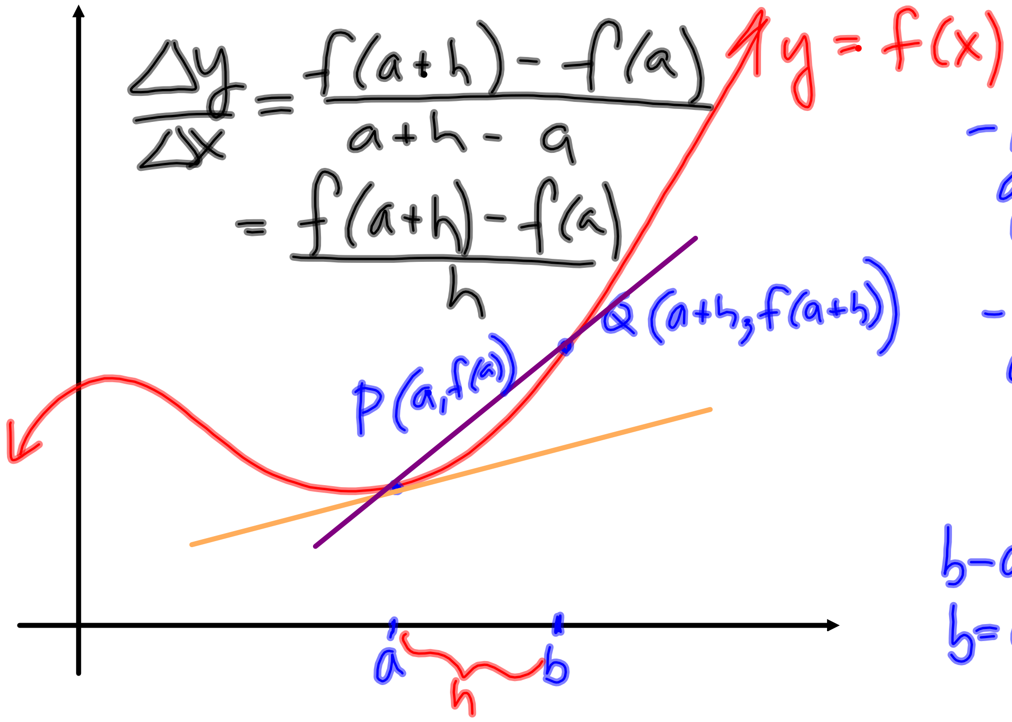


Definion:

An instantaneous rate of change is the rate of change measured at a single point on a connuous curve. Instantaneous rates of change correspond to the slope of the tangent line at that point.

The rate of change of $y = f(x)$ at a specific point $x = a$.

The Difference Quoent



- let P be a point on $y=f(x)$
- let Q be a point on $y=f(x)$

$b-a=h$
$b=a+h$

<u>The difference quoent:</u>

If $P(a, f(a))$ and $Q(a+h, f(a+h))$ are two points on the graph of $y = f(x)$, then the instantaneous rate of change of y with respect to x at P can be esmated using $\frac{\Delta y}{\Delta x} = \frac{f(a+h)-f(a)}{h}$ where h is a very small number.

Example 1) Instantaneous rates of change from a table

The following table shows the temperature of an oven as it warms from room temperature to 400°F. Esmate the instantaneous rate of change at exactly 5 minutes, given the following data:

Time (min) t	0	1	2	3	4	5	6	7	8	9	10
Temperature (°F) T	70	125	170	210	250	280	310	335	360	380	400

$T(t)$

• Choose some centred intervals around 5 min

Use interval $2 \leq t \leq 8$

$$\frac{\Delta T}{\Delta t} = \frac{360-170}{8-2} = 31.67\,°F/min$$

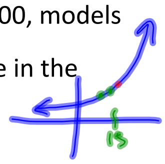
Use interval $4 \leq t \leq 6$

$$\frac{\Delta T}{\Delta t} = \frac{310-250}{6-4} = 30\,°F/min$$

$\therefore$ as the centre interval around 5, decreases it appears that the average R.o.C. gets closer to 30

Example 1) Instantaneous rates of change from an equaon

The populaon of a small town appears to be growing exponenally. Town planners think that the equaon $P(t)=35000(1.05)^t$ where $P(t)$ is the number of people in the town and t is the number of years aer 2000, models the size of the populaon. Esmate the instantaneous rate of change in the populaon in 2015.



Method 1: Interval Strategy 1

$14 \leq x \leq 15$ if $h=1$

- Preceding Interval: An interval of the independent variable of the form $a-h \leq x \leq a$ where h is a small posive value.
- Following interval: An interval of the independent variable of the form $a \leq x \leq a+h$ where h is a small posive value.

$h=1$

$15 \leq x \leq 16$

$a=15$, use preceding interval, $h=1$

$15-1 \leq t \leq 15$

$14 \leq t \leq 15$

$\frac{\Delta P}{\Delta t} = \frac{P(15)-P(14)}{15-14}$

$= 3464$ ppl/year

$h=0.5$

$15-0.5 \leq t \leq 15$

$14.5 \leq t \leq 15$

$\frac{\Delta P}{\Delta t} = \frac{P(15)-P(14.5)}{15-14.5}$

$= 3506$ ppl/year

As preceding interval $\to 0$, R.o.C $\uparrow$

Look at Following Interval.

when $h=1$

$15 \leq t \leq 15+1$

$15 \leq t \leq 16$

$\frac{\Delta P}{\Delta t} = \frac{P(16)-P(15)}{16-15}$

$= 3{,}639$ ppl/yr

when $h=0.5$

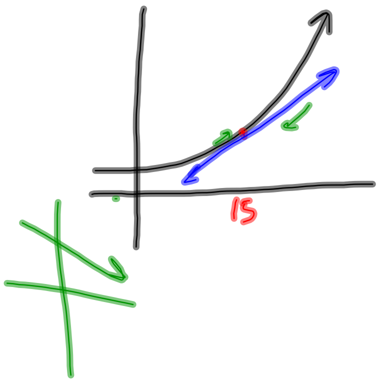
$15 \leq t \leq 15+.5$

$15 \leq t \leq 15.5$

$\frac{\Delta P}{\Delta t} = \frac{P(15.5)-P(15)}{15.5-15}$

$= 3594$ ppl/yr.

As the following interval $\to 0$, R.o.f.C $\downarrow$



Consider interval:

$14.5 \leq t \leq 15.5$

$= \frac{3506+3594}{2}$

$\boxed{= 3550 \text{ ppl/year}}$

Method 2: Interval Strategy 2

- Centered Interval: An interval of the independent variable of the form $a - h \leq x \leq a + h$ where h is a small posive value.

$a = 15$ choose $h = 0.5$

Centre interval: $14.5 \leq x \leq 15.5$

$$\frac{\Delta P}{\Delta t} = \frac{P(15.5) - P(14.5)}{15.5 - 14.5}$$

$$= \underline{\qquad\qquad}$$

Example 3) Instantaneous rates of change from an equaon

The volume of a cubic crystal, grown in a laboratory, can be modelled by $V(x) = x^3$

where *V(x)* is the volume measured in cubic cenmetres and *x* is the side length in cenmetres. Esmate the instantaneous rate of change in the crystal's volume with respect to its side length when the side length is 5 cm.

Soluon 1: Squeeze the centred intervals

$4.999 \leq x \leq 5.001$

$$\frac{\Delta V}{\Delta x} = \frac{V(5.001) - V(4.999)}{5.001 - 4.999}$$

$= \_\_\_\_\_\_\_\_\_\_$

$4.5 \leq x \leq 5.5$

R.of.C $= 75.25\ cm^3/cm$

$4.99 \leq x \leq 5.01$

R.of.C $= 75.001\ cm^3/cm$

Soluon 2: Use an algebraic approach and a general point

$V(x) = x^3 \quad , x = 5$

use the difference quotient.

$$\frac{\Delta V}{\Delta x} = \frac{V(5+h) - V(5)}{5+h-5}$$

$$= \frac{(5+h)^3 - 125}{h}$$

$h = -0.01$

$$= \frac{(5+(-0.01))^3 - 125}{-0.01}$$

$= 74.8501 \text{ cm}^3/\text{cm}$

$h = +0.01$

$$= \frac{(5+(0.01))^3 - 125}{0.01}$$

$= 75.1501 \text{ cm}^3/\text{cm}$

$$\text{estimate} = \frac{74.8501 + 75.1501}{2}$$

$= 75.001 \text{ cm}^3/\text{cm}.$

h/w: pg 71-73

B #3, 4, 6, 7 8, 9