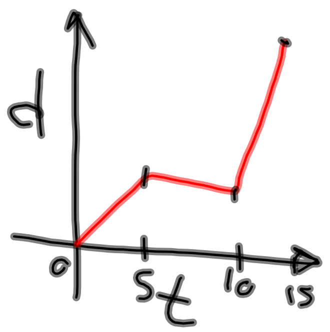
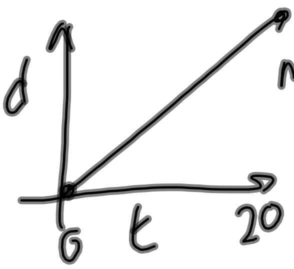
1.5: Slopes of Secants and Average Rate of Change

<u>Recall Definions:</u>

- Rate of Change: A measure of the change in one quanty (the dependent variable) with respect to a change in another quanty (the independent variable)
- Average Rate of Change: a change that takes place over an interval
- Instantaneous Rate of Change: a change that takes place in an instant





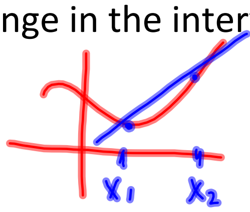
$m = \frac{y_2 - y_1}{x_2 - x_1}$

$$\boxed{m = \frac{\Delta y}{\Delta x}}$$

Theorem:

For a funcon, *f(x),* the average rate of change in the interval $x_1 \le x \le x_2$

is $\dfrac{\Delta y}{\Delta x} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$



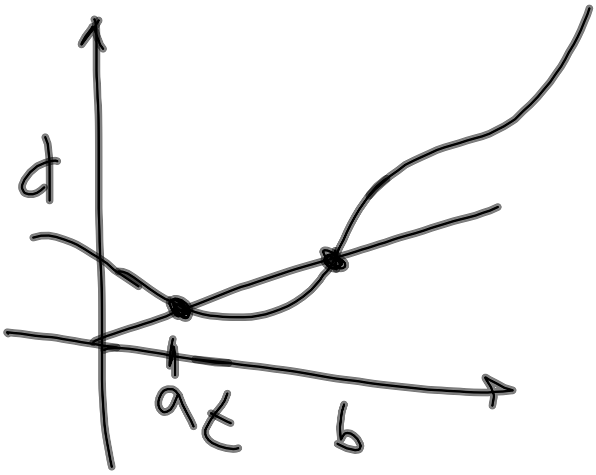
$x_1 < x < x_2$

Example 1

The following table represents the growth of bacteria populaon over a 10 hour period. During which 2 hour period did the populaon grow the fastest?

X	Y
0	850
2	1122
4	1481
6	1954
8	2377
10	3400

Time Interval (h)	Δb ($b_2 - b_1$) Change in number of Bacteria	Δt Change in me (t)	$\frac{\Delta b}{\Delta t}$ Average Rate of Change (bacteria/hour)
$0 \le t \le 2$	272	2	136
$2 \le t \le 4$	359	2	179.5
$4 \le t \le 6$	473	2	236.5
$6 \le t \le 8$	623	2	311.5
$8 \le t \le 10$	823	2	411.5


d
a
t
b

Example 1: Using a Graphical Model

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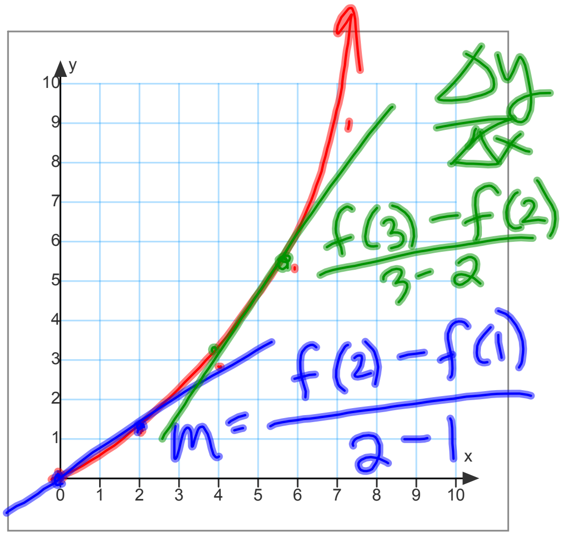
4

X	Y
o	850
2	1122
4	1481
6	1954
8	2577
10	3400



$\frac{\Delta y}{\Delta x}$

$\frac{f(3)-f(2)}{3-2}$

$m = \frac{f(2)-f(1)}{2-1}$

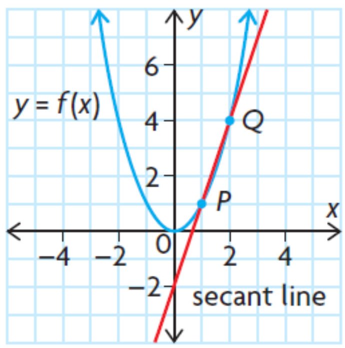
Secant Line: A line that passes through two points on the graph of a relaon.

Secant lines are used to determine the average rate of change over an interval.

$\frac{\Delta y}{\Delta x} = \frac{f(2)-f(1)}{2-1}$

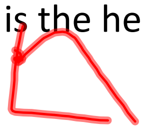
$= \frac{4-1}{2-1}$

$= \frac{3}{1}$



Example 2: Determining Average Rates of Change from an Equaon

A rock is tossed upward from a cliff that is 120 m above the water. The height of the rock above the water is modelled by $h(t) = -5t^2 + 10t + 120$ where *h(t)* is the height in metres and *t* is the me in seconds.



(a) Calculate the average rate of change in height during each of the following me intervals:

$0 \le t \le 1$	$1 \le t \le 2$	$2 \le t \le 3$	$3 \le t \le 4$
5m/s	-5 m/s	-15m/s	-25 m/s

(b) As the me increases, what do you noce about the average rate of change in height during each 1 s interval? What does this mean?

(c) Describe what the average rate of change means in this situaon.

$h(t) = -5t^2 + 10t + 120$

$-5 + 10 + 120$

(i) $0 \leq t \leq 1$

$= \dfrac{h(1) - h(0)}{1 - 0}$

$= \dfrac{125 - 120}{1}$

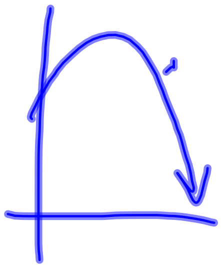
$= 5$ m/s

(ii) $1 \leq t \leq 2$

$= \dfrac{h(2) - h(1)}{2 - 1}$

$= \dfrac{120 - 125}{1}$

$= \dfrac{-5}{1} = -5$ m/s.



$h(t) = -5t^2 + 10t + 120$

(iii) $2 \leq t \leq 3$

$= \dfrac{h(3) - h(2)}{3 - 2}$

$=$

$= \dfrac{105 - 120}{1}$

$\dfrac{-15}{1}$

$= -15\text{m/s}$

(iv) $3 \leq t \leq 4$

$= \dfrac{h(4) - h(3)}{4 - 3}$

$= \dfrac{80 - 105}{1}$

$= \dfrac{-25}{1}$

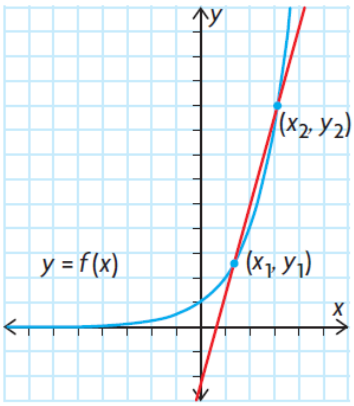
$= -25\text{m/s}$

-80

1.5: Average Rates of Change: Key Ideas

- A rate of change is a measure of how quickly one quanty (the dependent variable) changes with respect to another quanty (the independent variable)
- Average rates of change:
  > represent the rate of change over a specific interval

  > correspond to the slope of the secant between two points P_1 (x_1, y_1) and P_2 (x_2, y_2)



- An average rate of change can be determined by calculang the slope between two points given in a table of values or by using an equaon

p. 62 - 64

1-3, 5, 7, 12