# 1.2: Characteristics of Polynomial Functions

**Learning Goals:**

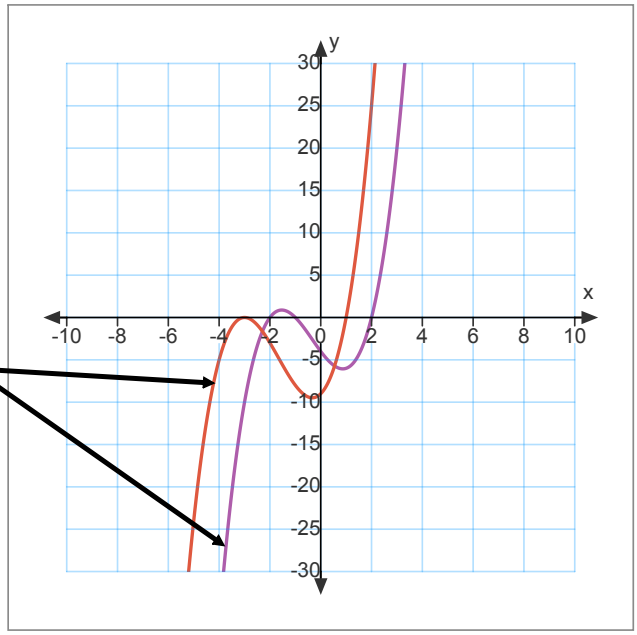
- identify the characteristics of the graphs and equations of general polynomial functions
- establish the relationship between finite differences and the equations of polynomial functions

# Investigating cubic functions - odd degree functions

$y = x^3$

$y = x^3 + x^2 - 4x - 4$

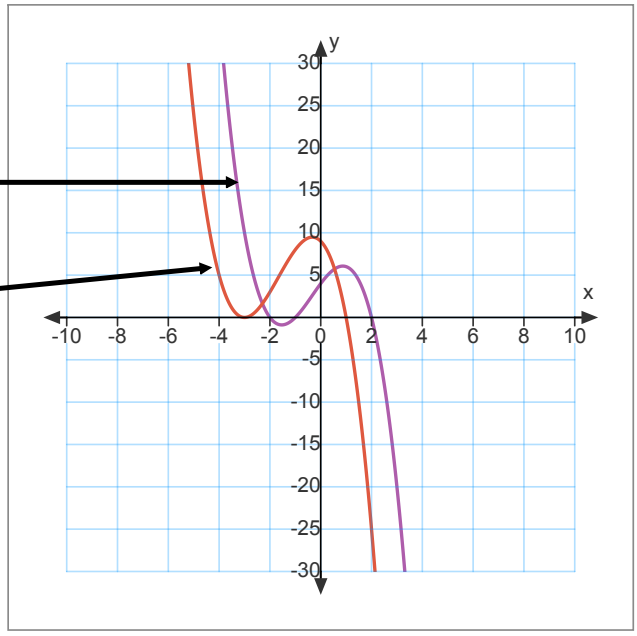
$y = x^3 + 5x^2 + 3x - 9$



$y = -x^3$

$y = -x^3 - x^2 + 4x + 4$

$y = -x^3 - 5x^2 - 3x + 9$

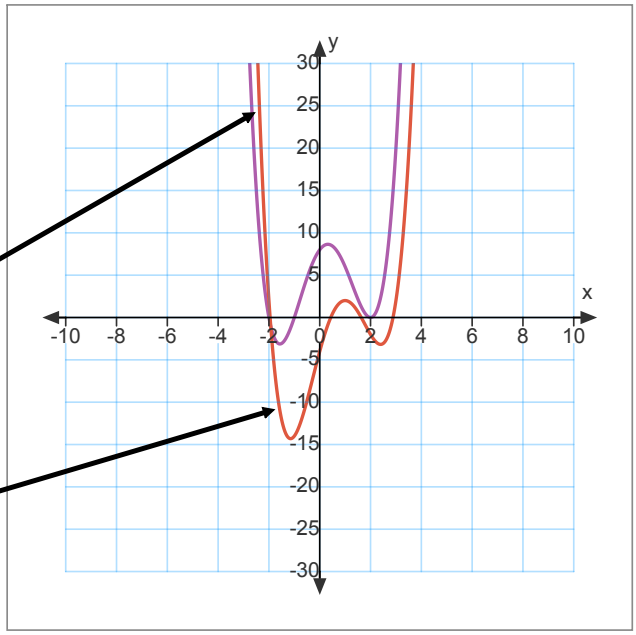


# Investigating quartic functions - even degree functions

$y = x^4$

$y = x^4 - x^3 - 6x^2 + 4x + 8$

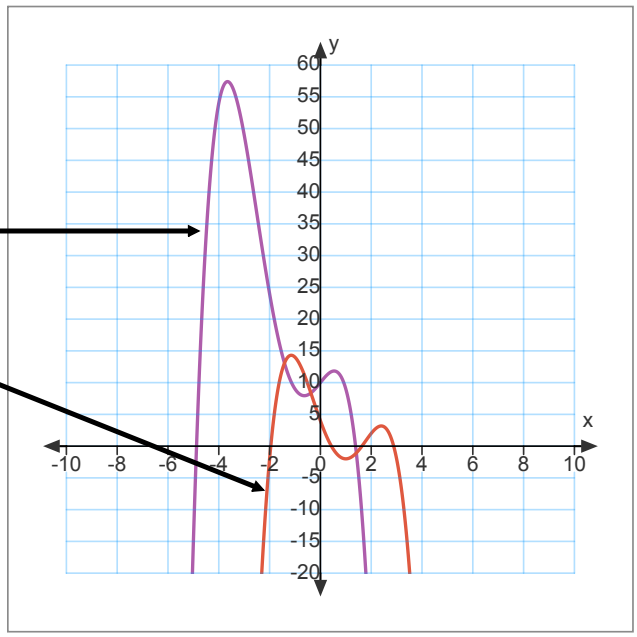
$y = x^4 - 3x^3 - 3x^2 + 11x - 4$



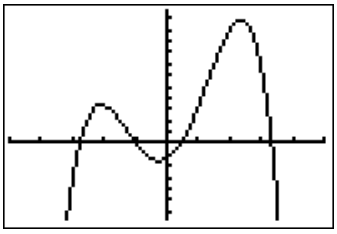
$y = -x^4$

$y = -x^4 - 5x^3 + 5x + 10$

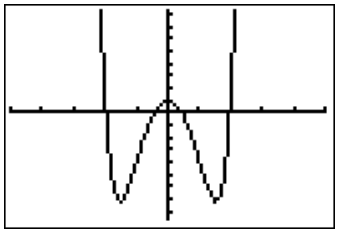
$y = -x^4 + 3x^3 + 3x^2 - 11x + 4$



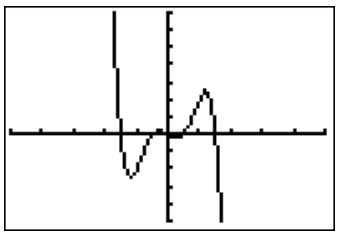
Example: Match a polynomial function with its graph



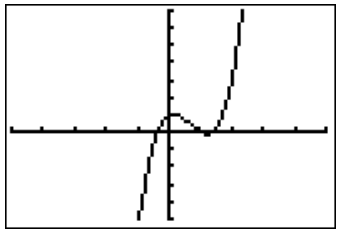
$y = -x^4 + 10x^2 + 5x - 4$



$y = x^6 - 16x^2 + 3$



$y = -2x^5 + 5x^3 - x$



$y = 2x^3 - 4x^2 + x + 1$

# Investigating Finite Differences

| Differences | | | | |
|---|---|---|---|---|
| x | y | First | Second | Third |
| -3 | -36 | | | |
| -2 | -12 | 24 | | |
| -1 | -2 | 10 | -14 | |
| 0 | 0 | 2 | -8 | 6 |
| 1 | 0 | 0 | -2 | 6 |
| 2 | 4 | 4 | 4 | 6 |
| 3 | 18 | 14 | 10 | 6 |
| 4 | 48 | 30 | 16 | 6 |

Constant 3rd differences, ∴ this polynomial function is of degree 3.

# Finite Differences

For a polynomial function of degree *n*, where *n* is a positive integer, the *n*th differences:

- are equal (constant)
- have the same sign as the leading co-efficient
- are equal to *a*[*n* x (*n* - 1) x ... x 2 x 1], where a is the leading coefficient

$[n \times (n-1) \times \ldots \times 2 \times 1] = n!$

So, $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Example:

Each table of values represents a polynomial function. Use finite differences to determine:

i) the degree of the polynomial function

ii) the sign of the leading coefficient

iii) the value of the leading coefficient

| x | y | 1st differences | 2nd differences | 3rd differences |
|---|---|---|---|---|
| -3 | -36 | | | |
| -2 | -12 | 24 | | |
| -1 | -2 | 10 | -14 | |
| 0 | 0 | 2 | -8 | 6 |
| 1 | 0 | 0 | -2 | 6 |
| 2 | 4 | 4 | 4 | 6 |
| 3 | 18 | 14 | 10 | 6 |
| 4 | 48 | 30 | 16 | 6 |

(i) Degree = 3 (b/c constant 3rd differences)

(ii) Sign of leading co-efficient is "+"

(iii) $6 = a(3!)$

$6 = (3 \times 2 \times 1)a$

$6 = 6a$

$1 = a$

∴ leading co-efficient is +1.

| x | y | 1st differences | 2nd differences | 3rd differences | 4th differences |
|---|---|---|---|---|---|
| -2 | -54 | | | | |
| -1 | -8 | 46 | | | |
| 0 | 0 | 8 | -38 | | |
| 1 | 6 | 6 | -2 | 36 | |
| 2 | 22 | 16 | 10 | 12 | -24 |
| 3 | 36 | 14 | -2 | -12 | -24 |
| 4 | 12 | -24 | -38 | -36 | -24 |
| 5 | -110 | -122 | -98 | -60 | -24 |

(i) Degree = 4

(ii) Sign of leading co-efficient is '-'

(iii) $-24 = 4!a$

$-24 = 24a$

$-1 = a$

$\therefore$ leading co-efficient is $-1$.